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SOVIET INSTRUMENTATION AND
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# Automation and Remote Control

(The Soviet Journal *Avtomatika i Telemekhanika* in English Translation)

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# A METHOD OF DETERMINING THE OPTIMAL PULSE RESPONSE FUNCTION FOR ONE CLASS OF DISTURBANCES

P. S. Matveev

(Moscow)

A method is presented for the determination of the optimum pulse response function for a servo system in the case when the system's input consists of given harmonic and exponential functions of time plus a stationary random function. In solving the problem, use is made of the relationship between the correlation function and Green's function. Two examples illustrate the method.

In a number of works (for example, [1-3]), there are obtained theoretical results related to the determination of optimal characteristics of dynamics systems which are subjected to random disturbances. These results are useful in those cases when the functions of time are given as polynomials of degree $r$, the solutions being obtained synthetically. Although reference is made in [2,4] to the possibility of using the aforementioned results when the disturbances are given in the form of sums of harmonic and exponential functions, no developed method has appeared in the literature for determining the pulse response function for these latter cases. The problem which the present paper proposes to solve is, using the mathematical relationship between the Greens function and the correlation function for a self-adjoint differential system (cf., for example, [5,6]), to give a method for determining the optimal pulse response function, given the requirements that the least possible mean-square error be attained by a system for which the correlation function of the signal and noise, and the duration of the transient response, are given. It is assumed that the signal may be represented in the time domain as the sum of given harmonic and exponential components. We shall consider the solution of the problem to be the obtaining of the reproduction operators:

$$H(p) = \sum_{i=0}^{n} H_i p^i.$$

## 1. Posing of the Problem

We shall assume that the system under consideration is acted upon by a controlling stimulus $y(t)$ and a disturbing stimulus $n(t)$. Let the controlling stimulus be the sum of two components, $y(t) = g(t) + m(t)$, where $g(t)$ is a given function of time in the form of a sum of harmonic and exponential functions and certain of their derivatives, and $m(t)$ is a stationary random function with a given correlation function $R_m(\tau)$ or a given spectral density $S_m(\omega)$.

The disturbing stimulus (noise) is also assumed to be a stationary random function whose correlation function $R_n(\tau)$ or spectral density $S_n(\omega)$ is given. For simplicity we shall assume that the functions $m(t)$ and $n(t)$ are uncorrelated and have mean values of zero. We shall assume that the quantity to be reproduced is the function

$$h(t) = H(p)\, y(t) \tag{1}$$

or

$$h(t) = \int_{-\infty}^{\infty} y(t-\tau)\, x(\tau)\, d\tau, \tag{2}$$

where

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H(j\omega)\, e^{j\omega t}\, d\omega. \tag{3}$$

We shall further assume that the correlation function $R(\tau)$ of the sum of the processes $m(t)$ and $n(t)$ has $2q$ continuous derivatives and, consequently, that the process $g(t) + m(t) + n(t)$ can be differentiated $q$ times. Then, the process $x(t)$ at the system's output may be written (cf. [6]) in the form

$$x(t) = \sum_{j=0}^{q} \int_{0}^{T} [g(t-\tau) + m(t-\tau) + n(t-\tau)]\, dW_j(\tau), \tag{4}$$

where the $W_j(\tau)$ are functions of bounded variation which characterize the optimal system. We shall assume that they are continuous within the interval of integration $(0, T)$ and have at most first-order discontinuities at $t = 0$ and $t = T$, such that the Stieltjes integrals in (4) exist. Integration of expression (4) by parts with zero initial conditions leads to

$$\begin{aligned} x(t) = &\int_{0}^{T} [g(t-\tau) + m(t-\tau) + n(t-\tau)]\, k(\tau)\, d\tau \\ &+ \left\{ \sum_{j=0}^{q} \frac{d^j}{d\tau^j} [g(t-\tau) + m(t-\tau) + n(t-\tau)]\, C_j \right\}_{\tau=0} \\ &+ \left\{ \sum_{j=0}^{q} \frac{d^j}{d\tau^j} [g(t-\tau) + m(t-\tau) + n(t-\tau)]\, D_j \right\}_{\tau=T}, \end{aligned} \tag{5}$$

where $k(\tau)$ is the pulse response function of the system for $0 < t < T$.

The system's error of reproduction may be written in the form

$$\begin{aligned} \varepsilon(t) = H(p)\, y(t) - x(t) = &\int_{-\infty}^{\infty} [g(t-\tau) + m(t-\tau)]\, x(\tau)\, d\tau - \\ &\int_{0}^{T} [g(t-\tau) + m(t-\tau) + n(t-\tau)]\, k(\tau)\, d\tau - \\ &\left\{ \sum_{j=0}^{q} \frac{d^j}{d\tau^j} [g\ (t-\tau) + m(t-\tau) + n(t-\tau)]\, C_j \right\}_{\tau=0} - \\ &\left\{ \sum_{j=0}^{q} \frac{d^j}{d\tau^j} [g(t-\tau) + m(t-\tau) + n(t-\tau)]\, D_j \right\}_{\tau=T}. \end{aligned} \tag{6}$$

We require here that the average value of reproduction error be identically zero, i.e.,

$$\begin{aligned} M[\varepsilon(t)] = &\int_{-\infty}^{\infty} g(t-\tau)\, x(\tau)\, d\tau - \int_{0}^{T} g(t-\tau)\, k(\tau)\, d\tau - \\ &- \sum_{j=0}^{q} \frac{d^j}{d\tau^j} g(t-\tau)\, C_j|_{\tau=0} - \sum_{j=0}^{q} \frac{d^j}{d\tau^j} g(t-\tau)\, D_j|_{\tau=T} = 0. \end{aligned} \tag{7}$$

We assume that in identity (7) $g(t) = \sum_{\nu=0}^{n} a_\nu P_\nu(t)$. Then, (7) can be rewritten in the form

$$\begin{aligned} &\int_{-\infty}^{\infty} \sum_{\nu=0}^{n} a_\nu P_\nu(t-\tau)\, x(\tau)\, d\tau \equiv \int_{0}^{T} \sum_{\nu=0}^{n} a_\nu P_\nu(t-\tau)\, k(\tau)\, d\tau \\ &+ \sum_{j=0}^{q} \frac{d^j}{d\tau^j} \sum_{\nu=0}^{n} a_\nu P_\nu(t-\tau)\, C_j|_{\tau=0} + \sum_{j=0}^{q} \frac{d^j}{d\tau^j} \sum_{\nu=0}^{n} a_\nu P_\nu(t-\tau)\, D_j|_{\tau=T}. \end{aligned} \tag{8}$$

We consider further the class of harmonic and exponential functions (sums of sines, cosines and exponentials) $P_\nu(t)$ which possess properties which permit the representation of $P_\nu(t-\tau)$ in the form

$$P_\nu(t-\tau)=\sum_{\mu=1}^{r} b_\mu^\nu(\tau)\,\psi_\mu^\nu(t), \tag{9}$$

where $\psi_\mu^\nu(t)$ and $b_\mu^\nu(\tau)$ $(\mu = 1, 2, \ldots, r)$ are linearly independent functions. By taking (9) into account, we may write identity (8) in the following way:

$$\int_{-\infty}^{\infty}\sum_{\nu=0}^{\eta} a_\nu \sum_{\mu=1}^{r} b_\mu^\nu(\tau)\,\psi_\mu^\nu(t)\,\varkappa(\tau)\,d\tau \equiv \int_0^T \sum_{\nu=0}^{\eta} a_\nu \sum_{\mu=}^{r} b_\mu^\nu(\tau)\,\psi_\mu^\nu(t)\,k(\tau)\,d\tau$$
$$+\sum_{j=0}^{}\frac{d^j}{d\tau^j}\sum_{\nu=0}^{\eta} a_\nu \sum_{\mu=1}^{r} b_\mu^\nu(\tau)\,\psi_\mu^\nu(t)\,C_j|_{\tau=0}+\sum_{j=0}^{}\frac{d^j}{d\tau^j}\sum_{\nu=0}^{\eta} a_\nu \sum_{\mu=1}^{r} b_\mu^\nu(\tau)\,\psi_\mu^\nu(t)\,D_j|_{\tau=T}. \tag{10}$$

Identity (10) defines $(\eta + 1)r$ constraints, imposed on the pulse response function by the transforming operator H(p). These constraints, as is easily seen, have the form:

$$Q_\mu^\nu=\int_0^T b_\mu^\nu(\tau)\,k(\tau)\,d\tau+\sum_{j=0}^{q}\frac{d^j}{d\tau^j}\,b_\mu^\nu(\tau)\,C_j|_{\tau=0}+$$
$$\sum_{j=0}^{q}\frac{d^j}{d\tau^j}\,b_\mu^\nu(\tau)\,D_{j\ \tau=T}\quad(\nu=0,\ 1,\ 2,\ \ldots,\ n;\ \mu=1,\ 2,\ \ldots,\ r) \tag{11}$$

and are constants.

For simplicity in the argument which follows, we let

$$\int_0^T [g(t-\tau)+m(t-\tau)+n(t-\tau)]\,\delta^{(j)}(\tau)\,d\tau = g^{(j)}(t)+m^{(j)}(t)+n^{(j)}(t) \tag{12}$$

and

$$\int_0^T [g(t-\tau)+m(t-\tau)+n(t-\tau)]\,\delta^{(j)}(T-\tau)\,d\tau =$$
$$g^{(j)}(t-T)+m^{(j)}(t-T)+\eta^{(j)}(t-T). \tag{13}$$

Taking (12) and (13) into account, we rewrite (10) and (11) as

$$\int_0^T \sum_{\nu=0}^{\eta} a_\nu \sum_{\mu=1}^{r} b_\mu^\nu(\tau)\,\psi_\mu^\nu\ (t)\ \left[k(\tau)+\sum_{j=0}^{q} C_j\,\delta^j(\tau)+\sum_{j=0}^{q} D_j\,\delta^j(T-\tau)\right]d\tau \equiv$$
$$\int_{-\infty}^{\infty}\sum_{\nu=0}^{\eta} a_\nu \sum_{\mu=1}^{r} b_\mu^\nu(\tau)\,\psi_\mu^\nu(t)\,\varkappa(\tau)\,d\tau \tag{14}$$

and

$$Q_\mu^\nu=\int_0^T b_\mu^\nu(\tau)\left[k(\tau)+\sum_{j=0}^{q} C_j\,\delta^{(j)}(\tau)+\sum_{j=0}^{q} D_j\delta^{(j)}(T-\tau)\right]d\tau. \tag{15}$$

Thus, for example, if $g(t) = a_0 \cos \omega_0 t$, $H(p) = H_0 + H_1 p$ and $q < 0$ then, based on identity (14), one may write

$$\int_{-\infty}^{\infty}[a_0\cos\omega_0\tau\cos\omega_0 t+a_0\sin\omega_0\tau\sin\omega_0 t]\,[H_0\delta(\tau)+H_1\delta'(\tau)]\,d\tau =$$
$$H_0[a_0\cos\omega_0\tau\cos\omega_0 t+a_0\sin\omega_0\tau\sin\omega_0 t]\,|_{\tau=0}+$$
$$H_1[-a_0\omega_0\cos\omega_0 t\sin\omega_0\tau+a_0\omega_0\sin\omega_0 t\cos\omega_0\tau]|_{\tau=0}\equiv \tag{16}$$

(continued)

$$\equiv \int_0^T [a_0 \cos \omega_0 \tau \cos \omega_0 t + a_0 \sin \omega_0 \tau \sin \omega_0 t]\, k(\tau)\, d\tau \equiv \tag{16}$$

$$a_0 \cos \omega_0 t \int_0^T \cos \omega_0 \tau\, k(\tau)\, d\tau + a_0 \sin \omega_0 t \int_0^T \sin \omega_0 \tau\, k(\tau)\, d\tau$$

or

$$H_0 = \int_0^T \cos \omega_0 \tau k(\tau)\, d\tau, \quad H_1 \omega_0 = \int_0^T \sin \omega_0 \tau k(\tau)\, d\tau. \tag{17}$$

In the example given we obtained two constraining conditions, such that $\eta + 1 = 1$ and $r = 2$ and, consequently, $(\eta + 1)r = 2$. If, in this case, we set $H_0 = 0$, the constraining conditions then take the form:

$$\int_0^T \cos \omega_0 \tau k(\tau)\, d\tau = 0, \qquad \int_0^T \sin \omega_0 \tau k(\tau)\, d\tau = H_1 \omega_0. \tag{18}$$

As a second example, we consider the case when $g(t) = a_0 \sin \omega_0 t + a_1 \cos 2\omega_1 t$, $H(p) = H_0$ and $q < 0$. For this example, identity (14) is written as follows:

$$\int_{-\infty}^{\infty} H_0 [a_0 \sin \omega_0 t \cos \omega_0 \tau - a_0 \cos \omega_0 t \sin \omega_0 \tau + a_1 \cos 2\omega_1 t \cos 2\omega_1 \tau +$$

$$a_1 \sin 2\omega_1 t \sin 2\omega_1 \tau]\, \delta(\tau)\, d\tau \equiv \int_0^T a_0 [\sin \omega_0 t \cos \omega_0 \tau - \tag{19}$$

$$\cos \omega_0 t \sin \omega_0 \tau]\, k(\tau) d\tau + a_1 \int_0^T [\cos 2\omega_1 t \cos 2\omega_1 \tau + \sin 2\omega_1 t \sin 2\omega_1 \tau]\, k(\tau)\, d\tau$$

or

$$H_0 = \int_0^T \cos \omega_0 \tau k(\tau)\, d\tau, \quad 0 = \int_0^T \sin \omega_0 \tau k(\tau)\, d\tau,$$

$$H_0 = \int_0^T \cos 2\omega_1 \tau k(\tau)\, d\tau, \quad 0 = \int_0^T \sin 2\omega_1 \tau k(\tau)\, d\tau. \tag{20}$$

In this case, we obtain four constraints, since $(\eta + 1) = 2$, $r = 2$, and $(\eta + 1)r = 4$.

We now consider a third example, wherein $g(t) = e^{-\alpha t}$, $H(p) = H_0$, and $q < 0$. For this example we may write

$$\int_{-\infty}^{\infty} H_0 [e^{-\alpha t} e^{\alpha \tau}]\, \delta(\tau)\, d\tau = \int_0^T e^{-\alpha t} e^{\alpha \tau} k(\tau)\, d\tau,$$

and, consequently, we get one constraining condition [since here $(\eta + 1) = 1$ and $r = 1$]:

$$H_0 = \int_0^T e^{\alpha \tau} k(\tau)\, d\tau.$$

Finally, we consider the example where

$$g(t) = a_0 \cos \omega_0 t, \quad H(p) = H_0 \ \& \ q = 0.$$

For this example, one may write

$$\int_{-\infty}^{\infty} H_0 [a_0 \cos \omega_0 t \cos \omega_0 \tau + a_0 \sin \omega_0 t \sin \omega_0 \tau] \delta(\tau)\, d\tau =$$

$$\int_0^T [a_0 \cos \omega_0 t \cos \omega_0 \tau + a_0 \sin \omega_0 t \sin \omega_0 \tau] [k(\tau) + C_0 \delta(\tau) + D_0 \delta(\tau - T)] d\tau =$$

$$a_0 \sin \omega_0 t \int_0^T \sin \omega_0 \tau [k(\tau) + C_0 \delta(\tau) + D_0 \delta(\tau - T)]\, d\tau +$$

$$a_0 \cos \omega_0 t \int_0^T \cos \omega_0 \tau [k(\tau) + C_0 \delta(\tau) + D_0 \delta(\tau - T)]\, d\tau$$

or

$$H_0 = \int_0^T \cos \omega_0 \tau [k(\tau) + C_0 \delta(\tau) + D_0 \delta(\tau - T)]\, d\tau,$$

$$0 = \int_0^T \sin \omega_0 \tau [k(\tau) + C_0 \delta(\tau) + D_0 \delta(\tau - T)]\, d\tau.$$

Here, $(\eta + 1) = 1$, $r = 2$ and $(\eta + 1)r = 2$.

## 2. Conditions for a Minimum Mean-Square Error

We write the expression for the error as

$$\varepsilon(t) = H(p)\, y(t) - \int_0^T [g(t-\tau) + m(t-\tau) + n(t-\tau)] \times$$

$$\left[k(\tau) + \sum_{j=0}^{q} C_j \delta^{(j)}(\tau) + \sum_{j=0}^{q} D_j \delta^{(j)}(\tau - T)\right] d\tau.$$

Then, if identity (14) is satisfied, we have

$$\varepsilon(t) = H(p)\, m(t) - \int_0^T [m(t-\tau) + n(t-\tau)] \times$$

$$\left[k(\tau) + \sum_{j=0}^{q} C_j \delta^{j}(\tau) + \sum_{j=0}^{q} D_j \delta^{j}(\tau - T)\right] d\tau. \tag{21}$$

If we square the right and left members of expression (21), we obtain

$$\varepsilon^2(t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} m(t-\theta)\, m(t-\tau) \varkappa(\theta) \varkappa(\tau)\, d\theta\, d\tau -$$

$$- 2 \int_{-\infty}^{\infty} \int_0^T m(t-\theta) [m(t-\tau) + n(t-\tau)] \varkappa(\theta) \times$$

$$\times \left[k(\tau) + \sum_{j=0}^{q} C_j \delta^{(j)}(\tau) + \sum_{j=0}^{q} D_j \delta^{(j)}(\tau - T)\right] d\theta\, d\tau + \int_0^T \int_0^T [m(t-\theta) + n(t-\theta)] \times \tag{22}$$

$$\times [m(t-\tau) + n(t-\tau)] \left[k(\theta) + \sum_{j=0}^{q} C_j \delta^{(j)}(\theta) + \sum_{j=0}^{q} D_j \delta^{(j)}(\theta - T)\right] \times$$

$$\times \left[k(\tau) + \sum_{j=0}^{q} C_j \delta^{(j)}(\tau) + \sum_{j=0}^{q} D_j \delta^{(j)}(\tau - T)\right] d\theta\, d\tau$$

and, consequently,

$$M[\varepsilon^2(t)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} R_m(\tau-\theta)\varkappa(\tau)\varkappa(\theta)\,d\tau\,d\theta -$$

$$-2\int_{-\infty}^{\infty}\int_{0}^{T} R_m(\tau-\theta)\varkappa(\tau)\Big[k(\theta)+\sum_{j=0}^{q}C_j\delta^{(j)}(\theta)+\sum_{j=0}^{q}D_j\delta^{(j)}(\theta-T)\Big]d\tau\,d\theta +$$

$$+\int_{0}^{T}\int_{0}^{T} R(\tau-\theta)\Big[k(\tau)+\sum_{j=0}^{q}C_j\delta^{(j)}(\tau)+\sum_{j=0}^{q}D_j\delta^{(j)}(\tau-T)\Big]\times \tag{23}$$

$$\times\Big[k(\theta)+\sum_{j=0}^{q}C_j\delta^{(j)}(\theta)+\sum_{j=0}^{q}D_j\delta^{(j)}(\theta-T)\Big]d\tau\,d\theta,$$

where $R(\tau-\theta)=R_m(\tau-\theta)+R_n(\tau-\theta)$.

The problem is now to find a pulse response function k(t) which simultaneously minimizes $M[\epsilon^2(t)]$ and satisfies conditions (15). To solve this problem, as is well known, it is necessary to set up the expression

$$I_k = M[\varepsilon^2(t)] - 2\sum_{\nu=0}^{\eta}\sum_{\mu=1}^{r}\gamma_\mu^\nu Q_\mu^\nu \tag{24}$$

and then to add to k(t) its variation $\Delta k(t)$.

For the result, we get

$$\Delta I_k = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} R_m(\tau-\theta)\varkappa(\tau)\varkappa(\theta)\,d\tau\,d\theta +$$

$$+\int_{0}^{T}\Big\{k(\tau)+\sum_{j=0}^{q}C_j\delta^{(j)}(\tau)+\sum_{j=0}^{q}D_j\delta^{(j)}(\tau-T)+\Delta\Big[k(\tau)+\sum_{j=0}^{q}C_j\delta^{(j)}(\tau)+$$

$$+\sum_{j=0}^{q}D_j\delta^{(j)}(\tau-T)\Big]\Big\}\Big\{\int_{0}^{T}\Big\{k(\theta)+\sum_{j=0}^{q}C_j\delta^{(j)}(\theta)+\sum_{j=0}^{q}D_j\delta^{(j)}(\theta-T)+ \tag{25}$$

$$+\Delta\Big[k(\theta)+\sum_{j=0}^{q}C_j\delta^{(j)}(\theta)+\sum_{j=0}^{q}D_j\delta^{(j)}(\theta-T)\Big]\Big\}R(\tau-\theta)\,d\theta-2P(\tau)\Big\}d\tau$$

$$0\leqslant t\leqslant T$$

or

$$\Delta I_k = M[\varepsilon^2(t)] + 2\Delta E_1 + \Delta^2 E_2, \tag{26}$$

where

$$E_1=\int_{0}^{T}\Big[k(\tau)+\sum_{j=0}^{q}C_j\delta^{(j)}(\tau)+\sum_{j=0}^{q}D_j\delta^{(j)}(\tau-T)\Big]d\tau\times$$

$$\Big\{\int_{0}^{T}R(\tau-\theta)\Big[k(\theta)+\sum_{j=0}^{q}C_j\delta^{(j)}(\theta)+\sum_{j=0}^{q}D_j\delta^{(j)}(\theta-T)\Big]d\theta-P(\tau)\Big\}, \tag{27}$$

$$E_2=\int_{0}^{T}\Big[k(\tau)+\sum_{j=0}^{q}C_j\delta^{(j)}(\tau)+\sum_{j=0}^{q}D_j\delta^{(j)}(\tau-T)\Big]d\tau\times$$

$$\int_{0}^{T}R(\tau-\theta)\Big[k(\theta)+\sum_{j=0}^{q}C_j\delta^{(j)}(\theta)+\sum_{j=0}^{q}D_j\delta^{(j)}(\theta-T)\Big]d\theta, \tag{28}$$

$$P(t)=\int_{-\infty}^{\infty}R_m(t-\tau)\varkappa(\tau)d\tau+\sum_{\nu=0}^{\eta}\sum_{\mu=1}^{r}\gamma_\mu^\nu b_\mu^\nu(t).$$

By differentiating (26) with respect to $\Delta$ and then setting $\Delta = 0$, we obtain the necessary and sufficient condition that $\Delta I_k$ be a minimum in the form $E_1 = 0$, or

$$\int_0^T R(t-\tau)\left[k(\tau)+\sum_{j=0}^{q} C_j \delta^{(j)}(\tau)+\sum_{j=0}^{q} D_j \delta^{j}(\tau-T)\right] d\tau - P(t) = 0 \quad (0 \leqslant t \leqslant T), \tag{29}$$

where $q = j - k - 1$, and $\underline{j}$ and $\underline{k}$ are defined by the expressions for the spectral densities.

## 3. Solution of the Integral Equation

The solution of integral Eq. (29) will be obtained for the class of stationary processes whose spectral densities can be expressed as rational fractions:

$$S(\omega) = \frac{a_0\omega^{2k} + a_1\omega^{2k-2} + \ldots + a_k}{b_0\omega^{2j} + b_1\omega^{2j-2} + \ldots + b_j} = \frac{M(\omega)\,M^*(\omega)}{L(\omega)\,L^*(\omega)}\,. \tag{30}$$

It is possible to show (cf. [5]) that the correlation function of a stationary process which has a spectral density of the form of (30) is related to the Greens function of the self-conjugate boundary problem

$$\begin{gathered} L(p)\,L^*(p)\,y(t) = 0, \\ \lim_{t\to\infty} y^{(n)}(t) = \lim_{t\to\infty} y^{(n)}(t) \quad (n = 0, 1, \ldots, j-1) \end{gathered} \tag{31}$$

by the relationship

$$R(t-\tau) = M(p)\,M^*(p)\,H(t-\tau), \tag{32}$$

where $H(t-\tau)$ is the Greens function of the self-conjugate differential system of (31), i.e., the solution of the equation

$$L(p)\,L^*(p)\,H(t-\tau) = \delta(t-\tau), \tag{33}$$

and the operators $M(p)$ and $L(p)$ equal the $M(\omega)$ and $L(\omega)$ of (30) for $p = j\omega$, and may be easily determined from the expressions for the spectral densities. Taking (32) into account, we rewrite integral Eq. (29) in the form

$$\begin{aligned} &\int_0^T M(p)\,M^*(p)\,H(t-\tau)\left[k(\tau)+\sum_{j=0}^{q} C_j\delta^{(j)}(\tau)+\sum_{j=0}^{q} D_j\delta^{(j)}(\tau-T)\right] d\tau = \\ &\qquad = \sum_{\nu=0}^{\eta}\sum_{\mu=1}^{r} \gamma_\mu^\nu\, b_\mu^\nu(t) + \int_{-\infty}^{\infty} R_m(t-\tau)\,x(\tau)\,d\tau \end{aligned}$$

or

$$\begin{aligned} &M(p)\,M^*(p)\left\{\int_0^T H(t-\tau)\left[k(\tau)+\sum_{j=0}^{q} C_j\delta^{(j)}(\tau)+\sum_{j=0}^{q} D_j\delta^{(j)}(\tau-T)\right] d\tau\right\} = \\ &\qquad = \sum_{\nu=0}^{\eta}\sum_{\mu=1}^{r} \gamma_\mu^\nu\, b_\mu^\nu(t) + \int_{-\infty}^{\infty} R_m(t-\tau)\,x(\tau)\,d\tau. \end{aligned} \tag{34}$$

The left member of the linear nonhomogeneous differential equation obtained has constant coefficients and is of order $2k$. If we denote the integral in the left member of (34) by $N(t)$, we get

$$M(p)\,M^*(p)\,N(t) = \sum_{\nu=0}^{\eta}\sum_{\mu=1}^{r} \gamma_\mu^\nu\, b_\mu^\nu(t) + \int_{-\infty}^{\infty} R_m(t-\tau)\,x(\tau)\,d\tau. \tag{35}$$

On the one hand, $N(t)$ is determined as the general solution of the differential equation which has the form:

$$N(t) = \sum_{n=1}^{2k} B_{2n}\, e^{\lambda_{2n} t} + M^{-1}(p)M^{*-1}(p)\left[\sum_{\nu=0}^{\eta}\sum_{\mu=1}^{r} \gamma_\mu^\nu\, b_\mu^\nu(t) + \int_{-\infty}^{\infty} R_m(t-\tau)\,x(\tau)\,d\tau\right], \tag{36}$$

and, on the other hand,

$$N(t) = \int_0^T H(t-\tau)\left[k(\tau) + \sum_{j=0}^{q} C_j \delta^{(j)}(\tau) + \sum_{j=0}^{q} D_j \delta^{(j)}(\tau - T)\right] d\tau, \tag{37}$$

where the $\lambda_{2n}$ are the roots of the equation

$$M(\lambda)\, M^*(\lambda) = 0. \tag{38}$$

If we apply the operator $L(p)L^*(p)$ to both members of (37), we obtain, on the basis of (33),

$$\int_0^T \delta(t-\tau)\left[k(\tau) + \sum_{j=0}^{q} C_j \delta^{(j)}(\tau) + \sum_{j=0}^{q} D_j \delta^{(j)}(\tau - T)\right] d\tau =$$

$$= L(p)\, L^*(p)\, N(t) \qquad (0 < t < T),$$

i.e.,

$$k(t) = L(p)\, L^*(p) N(t) \qquad (0 < t < T) \tag{39}$$

In expanded form,

$$k(t) = \sum_{n=1}^{2k} B_{2n}\, L(\lambda)\, L^*(\lambda)\, e^{\lambda_{2n} t} +$$

$$+ L(p)\, L^*(p)\, M^{-1}(p) M^{*-1}(p)\left[\sum_{\nu=0}^{\eta} \sum_{\mu=1}^{r} \gamma_\mu^\nu\, b_\mu^\nu(t) + \right. \tag{40}$$

$$\left. + \int_{-\infty}^{\infty} R_m(t-\tau) \times (\tau)\, d\tau\right] \qquad (0 < t < T).$$

In expression (40), terms of the form

$$L(p)\, L^*(p)\, M^{-1}(p)\, M^{*-1}(p) \sum_{\nu=0}^{\eta} \sum_{\mu=1}^{r} \gamma_\mu^\nu\, b_\mu^\nu(t) =$$

$$= \sum_{\nu=0}^{\eta} \sum_{\mu=1}^{r} \gamma_\mu^\nu\, L(p)\, L^*(p)\, M^{-1}(p)\, M^{*-1}(p)\, b_\mu(t) =$$

$$= \sum_{\nu=0}^{\eta} \sum_{\mu=1}^{r} \gamma_\mu^\nu\, D_\mu^\nu\, L(p)\, L^*(p)\, b_\mu^\nu(t) = \sum_{\nu=0}^{\eta} \sum_{\mu=1}^{r} \lambda_\mu^\nu\, L(p)\, L^*(p)\, b_\mu^\nu(t),$$

and, since

$$\frac{L(p)\, L^*(p)}{M(p)\, M^*(p)}\, e^{j\omega t} = \frac{|L(j\omega)|^2}{|M(j\omega)|^2}\, e^{j\omega t}$$

and

$$\frac{L(p)\, L^*(p)}{M(p)\, M^*(p)}\, e^{-j\omega t} = \frac{|L(j\omega)|^2}{|M(j\omega)|^2}\, e^{-j\omega t},$$

then, with account taken of the discontinuities at the points t = 0 and t = T, we can write the final expression for k(t) in the closed form:

$$k(t) = \sum_{n=1}^{2k} B_{2n} e^{\lambda_{2n} t} + \sum_{\nu=0}^{\eta} \sum_{\mu=1}^{r} A_\mu^\nu\, b_\mu^\nu(t) +$$

$$+ L(p)\, L^*(p)\, M^{-1}(p)\, M^{*-1}(p) \int_{-\infty}^{\infty} R_m(t-\tau) \times (\tau)\, d\tau + \tag{41}$$

$$+ \sum_{j=0}^{q} C_j \delta^{(j)}(t) + \sum_{j=0}^{q} D_j \delta^{(j)}(t - T) \qquad (q \neq j - k - 1,\ 0 \leqslant t \leqslant T).$$

The arbitrary constants $B_l$ and $A_\mu^\nu$, as well as $C_j$ and $D_j$, are determined from integral Eq. (29) and from conditions (15).

Example 1. Let $g(t) = \cos \omega_0 t$, $H(p) = H_0$, $m(t) = 0$, and $S_n(\omega) = 2\alpha/(\alpha^2 + \omega^2)$.

In this case, $2k = 0$, $2j = 2$, $q = 0$, $M(p)M^*(p) = 2\alpha$, $L(p)L^*(p) = \alpha^2 - p^2$, $\eta + 1 = 1$, $r = 2$, and $P(t-\tau) = \cos \omega_0 t \cos \omega_0 \tau + \sin \omega_0 t \sin \omega_0 \tau$.

On the basis of expressions (35) and (41) we may write

$$2\alpha N(t) = \gamma_1^0 \cos \omega_0 t + \gamma_2^0 \sin \omega_0 t,$$

i.e.,

$$N(t) = \frac{\gamma_1^0}{2\alpha} \cos \omega_0 t + \frac{\gamma_2^0}{2\alpha} \sin \omega_0 t \tag{42}$$

and

$$k(t) = (\alpha^2 - p^2)\left[\frac{\gamma_1^0}{2\alpha} \cos \omega_0 t + \frac{\gamma_2^0}{2\alpha} \sin \omega_0 t\right] = \\ = A_0 \cos \omega_0 t + A_1 \sin \omega_0 t \quad (0 < t < T). \tag{43}$$

By substituting expression (43) in identity (14), we get

$$\int_{-\infty}^{\infty} H_0 (\cos \omega_0 t \cos \omega_0 \tau + \sin \omega_0 t \sin \omega_0 \tau)\, \delta(\tau)\, d\tau = \int_0^T (\cos \omega_0 \tau \cos \omega_0 t + \\ + \sin \omega_0 \tau \sin \omega_0 t)\, [A_0 \cos \omega_0 \tau + A_1 \sin \omega_0 \tau + C_0 \delta(\tau) + D_0 \delta(\tau - T)]\, d\tau.$$

Whence

$$H_0 = \int_0^T \cos \omega_0 \tau\, [A_0 \cos \omega_0 \tau + A_1 \sin \omega_0 \tau + C_0 \delta(\tau) + D\delta(\tau - T)]\, d\tau = \\ = A_0 \left[\frac{1}{2}\tau + \frac{1}{4\omega_0} \sin 2\omega_0 \tau\right]\Big|_0^T + \frac{A_1}{2\omega_0} \sin 2\omega_0 \tau \Big|_0^T + C_0 \cos \omega_0 \tau|_{\tau=0} + \\ + D_0 \cos \omega_0 \tau|_{\tau=T} = A_0 \left(\frac{T}{2} + \frac{1}{4\omega_0} \sin 2\omega_0 T\right) + \frac{A_1}{2\omega_0} \sin^2 \omega_0 T + C_0 + D_0 \cos \omega_0 T \tag{44}$$

and

$$0 = \int_0^T \sin \omega_0 \tau\, [A_0 \cos \omega_0 \tau + A_1 \sin \omega_0 \tau + C_0 \delta(\tau) + D_0 \delta(\tau - T)]\, d\tau = \\ = \frac{A_0}{2\omega_0} \sin^2 \omega_0 \tau \Big|_0^T + A_1 \left(\frac{1}{2}\tau - \frac{1}{4\omega_0} \sin 2\omega_0 \tau\right)\Big|_0^T + C_0 \sin \omega_0 \tau|_{\tau=0} + \\ + D_0 \sin \omega_0 \tau|_{\tau=T} = \frac{A_0}{2\omega_0} \sin^2 \omega_0 T + A_1 \left(\frac{T}{2} - \frac{1}{4\omega_0} \sin 2\omega_0 T\right) + D_0 \sin \omega_0 T. \tag{45}$$

By substituting the value of $k(t)$ from (43) into integral Eq. (29), we obtain

$$\int_0^T e^{-\alpha|t-\tau|}\, [A_0 \cos \omega_0 \tau + A_1 \sin \omega_0 \tau + C_0 \delta(\tau) + D_0 \delta(\tau - T)]\, d\tau = \\ = \gamma_1^0 \cos \omega_0 t + \gamma_2^0 \sin \omega_0 t. \tag{46}$$

With the change of variable $t - \tau = \theta$, we rewrite integral Eq. (46) in the form

$$\int_{t-T}^{0} e^{\alpha\theta}\, [A_0 \cos \omega_0 (t - \theta) + A_1 \sin \omega_0 (t - \theta)]\, d\theta + \int_0^t e^{-\alpha\theta}\, [A_0 \cos \omega_0 (t - \theta) + \\ + A_1 \sin \omega_0 (t - \theta)]\, d\theta + C_0 e^{-\alpha t} + D_0 e^{-\alpha(T-t)} = \tag{47}$$

(continued) $$= \left[A_1\omega_0 - A_0\alpha + C_0(\alpha^2+\omega_0^2)\right]\frac{e^{-\alpha t}}{\alpha^2+\omega_0^2} + \frac{2\alpha}{\alpha^2+\omega_0^2}(A_0\cos\omega_0 t + A_1\sin\omega_0 t) + \tag{47}$$

$$+ \frac{e^{-\alpha(T-t)}}{\alpha^2+\omega_0^2}[A_0\omega_0\sin\omega_0 T - A_0\alpha\cos\omega_0 T -$$

$$- A_1\alpha\sin\omega_0 T - A_1\omega_0\cos\omega_0 T + D(\alpha^2+\omega_0^2)] = \gamma_1^0\cos\omega_0 t + \gamma_2^0\sin\omega_0 t.$$

From this we obtain the following equations:

$$\frac{A_1\omega_0 - A_0\alpha + C_0(\alpha^2+\omega_0^2)}{\alpha^2+\omega_0^2} = 0,$$

$$\frac{A_0\omega_0\sin\omega_0 T - A_0\alpha\cos\omega_0 T - A_1\alpha\sin\omega_0 T - A_1\omega_0\cos\omega_0 T + D_0(\alpha^2+\omega_0^2)}{\alpha^2+\omega^2} = 0. \tag{48}$$

The joint solution of Eqs. (44), (45), (47), and (48) determines all the unknowns which enter into the pulse response function. Moreover, it follows from Eq. (47) that

$$\gamma_1^\circ = \frac{2\alpha A_0}{\alpha^2+\omega_0^2}, \qquad \gamma_2^\circ = \frac{2\alpha A_1}{\alpha^2+\omega_0^2}. \tag{49}$$

It is obvious that, account being taken of the discontinuities at the points t = 0 and t = T, the pulse response function can be presented as

$$k(t) = k(t)_{0<t<T} + \sum_{j=0}^{q} C_{(j)}\delta^{(j)}(t) + \sum_{j=0}^{q} D_{(j)}\delta^{(j)}(t-T) \quad (0\leqslant t\leqslant T) \tag{50}$$

and, consequently, in the example under consideration,

$$k(t) = A_0\cos\omega_0 t + A_1\sin\omega_0 t + C_0\delta(t) + D_0\delta(t-T) \qquad (0\leqslant t\leqslant T).$$

By using Eq. (50) we can find the expression for the mean-square error for m(t) = 0. With m(t) = 0, integral Eq. (29) goes over into the form

$$\int_0^T R_n(t-\tau\left[k(\tau + \sum_{j=0}^{q} C_j\delta^{(j)}(\tau + \sum_{j=0}^{q} D_j\delta^{(j)}(\tau - T)\right]d\tau = \sum_{\nu=0}^{\eta}\sum_{\mu=1}^{r}\gamma_\mu^\nu b_\mu^\nu(t) \tag{51}$$

$$(0\leqslant t\leqslant T).$$

Then

$$M[\varepsilon^2(t)] = \int_0^T\left[k(\tau) + \sum_{j=0}^{q} C_j\delta^{(j)}(\tau) + \sum_{j=0}^{q} D_j\delta^{(j)}(\tau - T)\right]d\tau\int_0^T R_n(t-\theta)\times$$

$$\times\left[k(\theta) + \sum_{j=0}^{q} C_j\delta^{(j)}(\theta) + \sum_{j=0}^{q} D_j\delta^{(j)}(\theta - T)\right]d\theta. \tag{52}$$

Whence, taking (51) into account, we obtain

$$M[\varepsilon^2(t)] = \int_0^T\left[k(\tau) + \sum_{j=0}^{q} C_j\delta^{(j)}(\tau) + \sum_{j=0}^{q} D_j\delta^{(j)}(\tau - T)\right]\sum_{\nu=0}^{\eta}\sum_{\mu=1}^{r}\gamma_\mu^\nu b_\mu^\nu(\tau)\,d\tau = \tag{53}$$

$$= \sum_{\nu=0}^{\eta}\sum_{\mu=1}^{r}\gamma_\mu^\nu\hat{b}_\mu^\nu \qquad (0\leqslant t\leqslant T).$$

In our example,

$$M[\varepsilon^2(t)] = \gamma_1^\circ H_0 = \frac{2\alpha A_0 H_0}{\alpha^2+\omega_0^2}. \tag{54}$$

Example 2. Let g(t) = $\cos\omega_0 t$, m(t) = 0, $S_n(\omega) = N^2$, H(p) = 1, and q < 0. It is obvious that

$$k(t) = A_0\cos\omega_0 t + A_1\sin\omega_0 t \quad (0\leqslant t\leqslant T). \tag{55}$$

Then,

$$\int_{-\infty}^{\infty} (\cos \omega_0 t \cos \omega_0 \tau + \sin \omega_0 t \sin \omega_0 \tau)\, \delta(\tau)\, d\tau =$$

$$= \int_0^T (\cos \omega_0 t \cos \omega_0 \tau + \sin \omega_0 t \sin \omega_0 \tau)\, k(\tau)\, d\tau$$

or

$$\int_0^T \cos \omega_0 \tau\, [A_0 \cos \omega_0 \tau + A_1 \sin \omega_0 \tau]\, d\tau = \frac{A_0 T}{2} + \frac{A_0}{4\omega_0} \sin 2\omega_0 T + \frac{A_1}{2\omega_0} \sin^2 \omega_0 T = 1 \tag{56}$$

and

$$\int_0^T \sin \omega_0 \tau\, [A_0 \cos \omega_0 \tau + A_1 \sin \omega_0 \tau]\, d\tau = \frac{A_0}{2\omega_0} \sin^2 \omega_0 T + \frac{A_1}{2} T - \frac{A_1}{4\omega_0} \sin 2\omega_0 T = 0.$$

By solving, simultaneously, the equations in (56), we get

$$A_0 = \frac{2\omega_0 (T\omega_0 - \sin \omega_0 T \cos \omega_0 T)}{T^2 \omega_0^2 - \sin^2 \omega_0 T}, \qquad A_1 = -\frac{2\omega_0 \sin^2 \omega_0 T}{T^2 \omega_0^2 - \sin^2 \omega_0 T}. \tag{57}$$

The transient response may be written in the form

$$x(t) = \int_0^t \cos \omega_0 (t - \tau)\, k(\tau)\, d\tau = \frac{A_1}{2} t \cos \omega_0 t + \left(\frac{A_1}{\omega_0} + A_2 t\right) \frac{\sin \omega_0 t}{2} \quad (0 \leqslant t \leqslant T). \tag{58}$$

Taking Formulas (55), (57), and (58) into account, we consider two cases.

First Case. Let T = 1 and $\omega_0 = 0.25$. Then $A_1 = 3.98$, $A_2 = -23.66$, and

$$k(t) = 3.98 \cos 0.25\, t - 23.66 \sin \omega_0 t, \tag{59}$$

and

$$x(t) = 1.99 \cos 0.25\, t + \sin 0.25\, t\, (7.96 - 11.83\, t). \tag{60}$$

Figure 1 gives the results computed from Formulas (59) and (60), and also the value of cos 0.25 t.

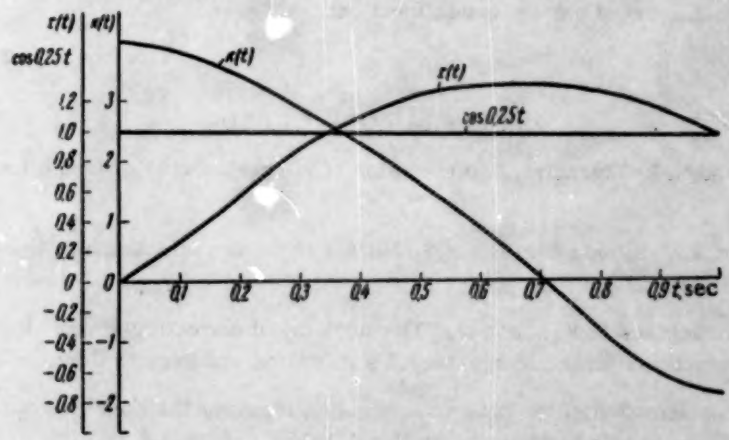


Fig. 1

Second Case. Let T = 1 and $\omega_0 = 5$. Then $A_1 = 2.19$, $A_2 = -0.38$, and

$$k(t) = 2.19 \cos 5\, t - 0.38 \sin 5\, t, \tag{61}$$

and

$$x(t) = 1.095 \cos 5\, t + (0.22 - 0.19\, t) \sin 5\, t. \tag{62}$$

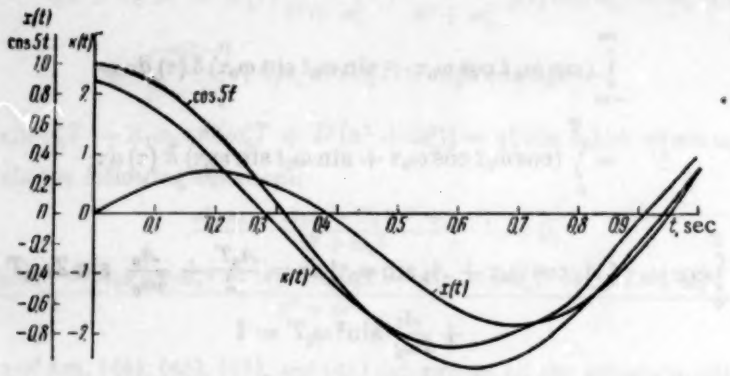


Fig. 2

Figure 2 gives the results computed from Formulas (61) and (62), and also the value of cos 5 t.

## SUMMARY

1. This paper presented a generalization of the method for obtaining the optimal pulse response function, as presented in [1], to the case when it is required to obtain the minimum mean-square error, given the correlation functions of the signal and the noise, the time functions in the form of sums of sines, cosines, and exponentials, and the duration of the transient response.

In [1-4], the question of choosing the time of observation T remained open. Here, as in [3], we have taken the time of observation to be the duration of the system's transient response to the signal g(t).

2. Application of this method may, in many cases, provide an essential simplification in the determination of the pulse response function, as compared to the method of work [1], since the approximations of sines and cosines may require polynomials of high degree. This fact could also lead to an increase in the mean-square error.

There is practical significance to such a problem in the case when it is necessary to filter signals which are represented on a finite interval by means of Fourier series with finite numbers of terms.

3. The examples herein provided show that there is no particular difficulty in applying the method presented here for the determination of pulse response functions.

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*See C. B. translation.

# APPROXIMATE DETERMINATION OF THE AUTO-OSCILLATIONS IN THE AUTOMATIC CONTROL SYSTEM FOR A SYNCHRONOUS MOTOR

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The auto-oscillations are investigated in the automatic control system for a synchronous motor, where the nonlinear portion is characterized by the functions $\theta^2 \sin\theta$, $(\sin\theta)^2 p\theta$, $(\sin\theta)^2 p^2\theta$, and $(\sin\theta)^2 p^3\theta$.

The automatic control of a synchronous motor (SM), working on a load, has for its purpose the maintenance of the best working regimens, from the standpoints of voltage oscillations and energy loss in the circuit, both of the SM itself and of the energy system supplying it.

As was shown in [1], underlying the best law for controlling the SM excitation is the principle of the constancy of reactive power output. It was shown in [2] that to implement such a control law as a function of the angular shift of the rotating axial field of the stator and rotor – the angle $\theta$ – it is necessary to introduce a nonlinear element – a squaring block – into the control system.

For the development of automated synchronous electrodrives for such machines as rolling mills, planers, broaching machines, grooving machines, power compressors, etc., for which the load varies from zero up to one, or one-and-a-half times the nominal moment, it is necessary to take the nonlinearity of the motors into account.

In our investigation, we shall take the following nonlinearities into account: the angular moment nonlinearity, of a sinusoidal nature, of the SM; the nonlinearity induced by the reaction of a synchronous machine's armature, and the nonlinearity of the squaring block introduced into the system to provide control of the SM under a constant output of reactive power.

An investigation of the periodic regimens of systems with nonlinear elements having sinusoidal characteristics was made by E. L. Urman [3] for the investigation of the synchronization conditions for synchronous machines.

The basis for our investigation is the principle of harmonic equilibrium of N. M. Krylov and N. N. Bogolyubov [4].

The block schematic for the system of automatic control of an SM is shown in Fig. 1. We will now set up the equations characterizing the processes in the nonlinear control system. The controlled object, the SM, is described by the following equations.

a) Equation of the SM's rotor motion:

$$\frac{mUE_d}{\omega_s X_d} \sin\theta + \frac{J}{p_m} p^2\theta = M_c, \qquad (1)$$

where the first term is the electromagnetic moment of the SM, m is the number of phases, U is the voltage in the supply network, $E_d$ is the idling emf, $\omega_s$ is the synchronized angular velocity, $X_d$ is the reactance along the axis, J is the moment of inertia applied to the SM's shaft, p is the number of pairs of poles of the SM, and $M_c$ is the load moment on the shaft.

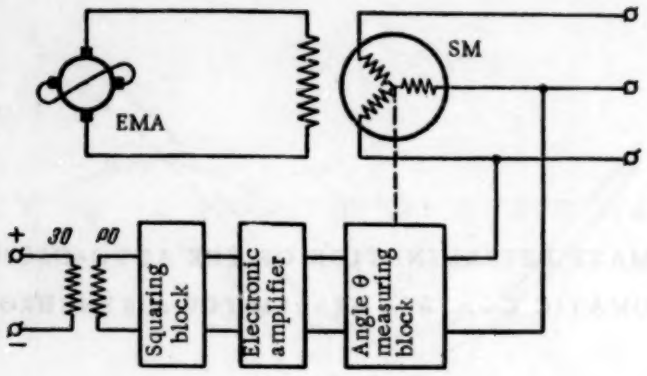


Fig. 1

b) The equations for the transient response in the excitation winding of the SM, neglecting magnetic saturation, and for the electromagnetic transient responses in the stator circuit:

$$E_{d_0} = E_d + T_{d_0} p E'_d, \quad E'_d = \frac{X'_d}{X_d} E_d + \frac{X_d - X'_d}{X_d} U \cos\theta, \tag{2}$$

where $E_{de}$ is the steady-state value of the idling emf, $E_{d_0}$ is the idling emf during the transient response, $E'_d$ is the emf of the SM during the reactance transients, $T_{d_0}$ is the idling time constant of the SM's excitation winding, and X is the transient reactance along the axis of the SM.

In correspondence with (2), we give the equation for the SM's emf in the following form:

$$E_{d_0} = (1 + T'_d p) E_d - T_{d_0} \frac{X_d - X'_d}{X_d} U \sin\theta \, p\theta \quad (T'_d = T_{d_0} X'_d / X_d). \tag{3}$$

The steady-state value of the emf, $E_{de}$, is related to the voltage at the input of the SM's excitation winding, $U_{in}$, by the relationship

$$E_{de} = k_m U_{in}, \tag{4}$$

where $k_m = X_{ad}/r_{ex}$; $X_{ad}$ is the reactance of the reaction armature along the axis of the SM, and $r_{ex}$ is the ohmic impedance of the excitation winding.

The equation of the measuring element is written in the form:

$$U_\theta = k_\theta \theta, \tag{5}$$

where $U_\theta$ is the output quantity and $k_\theta$ is the gain.

The equation for the squaring block, connected in series with a linear amplifier, has the form:

$$U_e = k_a k_{sq} U_\theta^2, \tag{6}$$

where $U_e$ is the magnitude at the output of the electronic amplifier, being simultaneously the input quantity to the EMA, and $k_{sq}$ is the gain of the squaring block.

With complete compensation for the armature reaction, the equation for the EMA will be:

$$(1 + T_{cw} p)(1 + T_s p) U_{in} = k_e U_e, \tag{7}$$

where $T_{cw}$ is the time constant of the control winding, $T_s$ is the time constant of the short-circuited network, and $k_e$ is the voltage gain.

Solving Eqs. (1), (3) and (4)-(7) simultaneously, we find

$$\alpha\theta^2 \sin\theta + \beta_1 (\sin\theta^2) p\theta + \beta_2 (\sin\theta)^2 p^2\theta + \beta_3 (\sin\theta)^3 p^3\theta + \\ + \frac{J}{p_m} \frac{p^2 (1 + T_{cw} p)(1 + T_s p)(1 + T'_d p)}{k_m k_e k_a k_{sq} k_\theta^2} \theta = \frac{(1 + T_{cw} p)(1 + T_s p)(1 + T'_d p)}{k_m k_e k_a k_{sq} k_\theta^2} M_c, \tag{8}$$

where

$$\alpha = \frac{mU}{\omega_s X_d}, \quad \beta_1 = \frac{m(X_d - X'_d)}{\omega_s X_d^2 k_m k_e k_a k_{sq} k_\theta^2} T_{d_0} U^2,$$

$$\beta_2 = \frac{m(X_d - X'_d)}{\omega_s X_d^2 k_m k_e k_a k_{sq} k_\theta^2} (T_{cw} + T_s) T_{d_0} U^2,$$

$$\beta_3 = \frac{m(X_d - X'_d)}{\omega_s X_d^2 k_m k_e k_a k_{sq} k_\theta^2} T_{cw} T_s T_{d_0} U^2.$$

We can put Eq. (8) into the following form:

$$\alpha y_1 + \beta_1 y_2 + \beta_2 y_3 + \beta_3 y_4 + \frac{J}{p_m} \frac{p^2 (1 + T_{oy} p)(1 + T_u p)(1 + T'_d p)}{k_m k_e k_a k_{sq} k_\theta^2} \theta = \frac{(1 + T_{oy} p)(1 + T_u p)(1 + T'_d p)}{k_m k_e k_a k_{sq} k_\theta^2} M_c, \tag{9}$$

where

$$y_1 = \theta^2 \sin\theta, \quad y_2 = (\sin\theta)^2 p\theta, \quad y_3 = (\sin\theta)^2 p^2\theta, \quad y_4 = (\sin\theta)^2 p^3\theta.$$

In seeking the auto-oscillations of the nonlinear system under consideration, we shall look for solutions in the form $\theta = \theta_0 + \theta_p = \theta_0 + a\sin\omega t$, where $\theta_0$ is the steady-state value of the angle and $a$ and $\omega$ are, respectively, the amplitude and frequency of the auto-oscillation being sought. With this, $p\theta = a\omega\cos\omega t$, $p^2\theta = -a\omega^2 \sin\omega t$ and $p^3\theta = -a\omega^3\cos\omega t$.

Letting $\omega t = U$, we have

$$\theta = \theta_0 + a\sin U, \quad p\theta = a\omega\cos U, \quad p^2\theta = -a\omega^2 \sin U, \quad p^3\theta = -a\omega^3\cos U. \tag{10}$$

By using the method of harmonic linearization [4] for the nonlinear functions entering into Eq. (9) (cf. the Appendix), we can separate out from (9) the equations for the periodic and the constant components.

The equation for the periodic components has the form:

$$\alpha q'(a, \theta_0)\theta_p + \alpha \frac{q'_1(a, \theta_0)}{\omega} p\theta_p + \beta_1 q''(a, \theta_0, \omega)\theta_p + \beta_1 \frac{q''_1(a, \theta_0, \omega)}{\omega} p\theta_p + \beta_2 \frac{q'''_1(a, \theta_0, \omega)}{\omega} p\theta_p + \beta_2 \frac{q_2(a, \theta_0, \omega)}{-\omega^2} p^2\theta_p + \beta_3 q^{IV}(a, \theta_0, \omega)\theta_p + \beta_3 \frac{q_3(a, \theta_0, \omega)}{-\omega^3} p^3\theta_p + \frac{J}{p_m} \frac{p^2(1 + T_{cw} p)(1 + T_s p)(1 + T'_d p)}{k_m k_e k_a k_{sq} k_\theta^2} \theta_p = 0. \tag{11}$$

The equation for the constant components has the form

$$\alpha q'_0(a, \theta_0) + \beta_1 q''_0(a, \theta_0, \omega) + \beta_2 q'''_0(a, \theta_0, \omega) + \beta_3 q^{IV}(a, \theta_0, \omega) + \frac{J}{p_m} \frac{p^2(1 + T_{cw} p)(1 + T_s p)(1 + T'_d p)}{k_m k_e k_a k_{sq} k_\theta^2} \theta_0 = \frac{(1 + T_{cw} p)(1 + T_s p)(1 + T'_d p)}{k_m k_e k_a k_{sq} k_\theta^2} M_c. \tag{12}$$

Equations (11) and (12) provide the possibility of obtaining the desired parameters of the auto-oscillation. However, in our case, the joint analytical solution of Eqs. (11) and (12) does not appear possible. Therefore, to determine the desired parameters of the auto-oscillation we employ, for some previously given value of $\theta_0$, a graphicoanalytic approximate method [5], using the Mikhailov stability criterion.

The characteristic equation corresponding to the equation for the periodic components has the form:

$$\beta_3 \frac{q_3(a, \theta_0, \omega)}{-\omega^3} p^3 + \beta_2 \frac{q_2(a, \theta_0, \omega)}{-\omega^2} p^2 + \left[ \alpha \frac{q_1'(a, \theta_0)}{\omega} + \beta_1 \frac{q_1''(a, \theta_0, \omega)}{\omega} + \right.$$

$$\left. + \beta_2 \frac{q_1'''(a, \theta_0, \omega)}{\omega} \right] p + [\alpha q'(a, \theta_0) + \beta_1 q''(a, \theta_0, \omega) + \beta_3 q^{IV}(a, \theta_0, \omega)] + \qquad (13)$$

$$+ \frac{J}{p_m} \frac{p^2 (1 + T_{cw} p)(1 + T_s p)(1 + T_d' p)}{k_m k_e k_a k_{sq} k_\theta^2} = 0.$$

We substitute p = jω in Eq. (13) and then, for each of several numerical values of ω, we construct the complex plot as a varies from zero to infinity. By using the method of successive approximations, we find those values of the frequency ω for which the origin of coordinates lies between two neighboring curves. Then, by constructing lines of identical amplitude, we isolate the region within which the origin of coordinates lies. Then, by the use of interpolation, we can determine the desired auto-oscillation parameters for the control system. It is obvious that the smaller the region inside of which the origin of coordinates lies, the more accurately are the auto-oscillation parameters determined.

The parameters thus obtained for the assumed auto-oscillation are investigated for stability by the use of the analytic relationship

$$\left(\frac{\partial x}{\partial a}\right)_{a=a_p} \left(\frac{\partial y}{\partial \omega}\right)_{\omega=\omega_p} - \left(\frac{\partial x}{\partial \omega}\right)_{\omega=\omega_p} \left(\frac{\partial y}{\partial a}\right)_{a=a_p} > 0,$$

characterizing stability according to the Mikhailov curves [5,6]. Here, x and y are, respectively, the real and imaginary parts of the Mikhailov curve, and $a_p$ and $\omega_p$ are, respectively, the amplitude and frequency of the auto-oscillation sought.

Since the oscillatory process in our control system is characterized by a fifth-degree equation (13), we must require, as an additional stability condition on the periodic solution found by us, that the Mikhailov curve, constructed in accordance with (13), successively traverse five quadrants in the counterclockwise sense.

If the automatic control system of the synchronous motor has a sufficiently large gain then, as follows from (8), the values of the coefficients $\beta_1$, $\beta_2$, and $\beta_3$ will be small, and the influence of the nonlinearity on the reaction of the synchronous machine's armature will be insignificant. With this, the characteristic equation, (13), can be expressed in the following form:

$$q'(a, \theta_0) + \frac{q_1'(a, \theta_0)}{\omega} p + \alpha' \frac{(1 + T_{cw} p)(1 + T_s p)(1 + T_d' p) p^2}{k_m k_e k_a k_{sq} k_\theta^2} = 0, \qquad (14)$$

where

$$\alpha' = \frac{J \omega_s X_d}{p_m m U}.$$

In this case, for given values of $\theta_0$ it is simpler to use the frequency method of L.S. Gol'dfarb [7] to determine the parameters of the auto-oscillation and to investigate their stability. The Gol'dfarb method makes it comparatively simple to gauge the effect of the flywheel mass on the synchronous motor's shaft and the control system's gain on the auto-oscillation.

In accordance with the method presented above, a trial computation was carried out to determine the auto-oscillation in the automatic control system for a synchronous motor characterized by the following data.

The power of the synchronous motor is P = 3.6 kilowatts, $k_m$ = 3.76, $T_{d_0}$ = 0.53 seconds, and $T_d'$ = 0.163 seconds.

The power of the electromachine amplifier is P = 4.5 kilowatts, $k_e$ = 10, $T_{cw}$ = 0.04 seconds, and $T_s$ = 0.1 second.

The total gain is $k_a k_{sq} k_\theta$ = 1. The flywheel moment of the synchronous electrodrive attached to the motor shaft was taken to be, in one case, 0.95 kilograms$^2$ and, in the other case, 300 kilograms$^2$. The steady-state values of the angle $\theta_0$ were taken equal to 15°, 25°, and 35°.

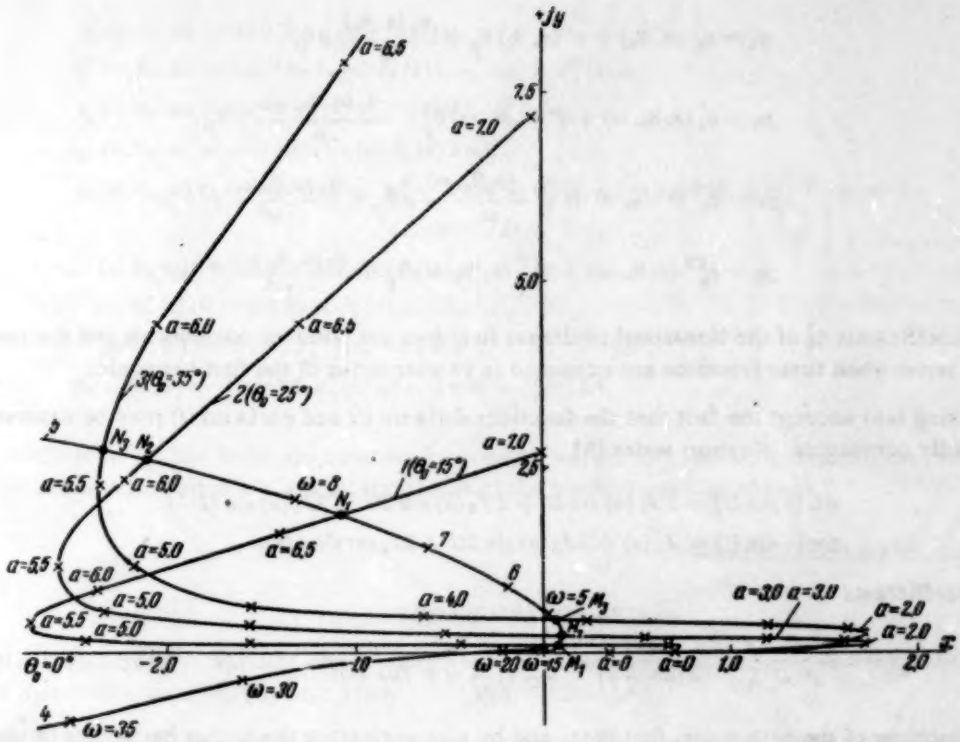


Fig. 2

As shown by the computations, there was no auto-oscillation in the automatic control system for $GD^2$ = = 0.95 kilograms$^2$. Fig. 2 shows no intersections between the amplitude characteristics of the nonlinear part (curves 1, 2 and 3) and the reciprocal amplitude-phase characteristic of the linear part (curve 4) and, consequently, there are no periodic regimens. For $GD^2$ = 300 kilograms$^2$, identical values for the auto-oscillation parameters were obtained using the methods of E. P. Popov and L. S. Gol'dfarb. On Fig. 2, the periodic solutions characterized by points $M_1$, $M_2$ and $M_3$ correspond to stable auto-oscillations in the control system, but the periodic solutions of points $N_1$, $N_2$ and $N_3$ characterize unstable oscillations. It is clear from Fig. 2 that with decreasing angle $\theta_0$ the amplitude characteristic of the nonlinear part contracts, and approximates to the axis of abscissas. In the limiting case when $\theta_0 = 0$, the amplitude characteristic of the nonlinear part coincides with the axis of abscissas from $-\infty$ to $+\infty$ as a varies from zero to infinity. In this case, there is always, for Eq. (14), an intersection of the amplitude characteristic of the nonlinear part (the axis of abscissas) with the reciprocal amplitude-phase characteristic of the linear part. Based on the previous investigation, the following conclusions may be drawn.

1. For small values of the angle $\theta_0$, the automatic control system for a synchronous motor is subject to auto-oscillations.

2. A significantly large flywheel mass on the shaft of the SM is capable of inducing auto-oscillations in the system, so that in developing automation for synchronous drives it is necessary to decrease the flywheel mass as much as possible. This step is very useful in decreasing the torsional oscillations of the motor shaft when the applied load varies periodically, and in avoiding damage to the shaft.

3. As the gain of the control system is increased, the auto-oscillations engendered by the nonlinearities considered here die out.

## Appendix

Using the method of N. M. Krylov and N. N. Bogolyubov, the harmonic linearization of the nonlinear functions $y_1 = \theta^2 \sin\theta$, $y_2 = (\sin\theta)^2 p\theta$, $y_3 = (\sin\theta)^2 p^2\theta$, and $y_4 = (\sin\theta)^2 p^3\theta$ is carried out in the following manner:

$$y_1 = q_0'(a, \theta_0) + q'(a, \theta_0)\theta_p + \frac{q_1'(a, \theta_0)}{\omega} p\theta_p,$$

$$y_2 = q_0''(a, \theta_0, \omega) + q''(a, \theta_0, \omega)\theta_p + \frac{q_1''(a, \theta_0, \omega)}{\omega} p\theta_p,$$

$$y_3 = q_0'''(a, \theta_0, \omega) + \frac{q_1'''(a, \theta_0, \omega)}{\omega} p\theta_p + \frac{q_2(a, \theta_0, \omega)}{-\omega^2} p^2\theta_p,$$

$$y_4 = q_0^{IV}(a, \theta_0, \omega) + q^{IV}(a, \theta_0, \omega)\theta_p + \frac{q_2(a, \theta_0, \omega)}{-\omega^2} p^2\theta_p.$$

The coefficients $q_1^j$ of the linearized nonlinear functions are constant components and the coefficients of the Fourier series when these functions are expanded in Fourier series of the first harmonic.

By taking into account the fact that the functions $\sin(a \sin U)$ and $\cos(a \sin U)$ may be expressed in the form of rapidly converging Neyman series [8]

$$\sin(a \sin U) = 2J_1(a) \sin U + 2J_3(a) \sin 3U + 2J_5(a) \sin 5U + \ldots,$$
$$\cos(a \sin U) = J_0(a) + 2J_2(a) \sin 2U + 2J_4(a) \sin 4U + \ldots,$$

where the coefficients

$$J_n(a) = \frac{a^n}{2^n n!}\left(1 - \frac{a^2}{2(2n+2)} + \frac{a^4}{2\times 4(2n+2)(2n+4)} - \ldots\right)(n = 0, 1, \ldots, \infty),$$

are Bessel functions of the n-th order, first type, and by also neglecting the higher harmonics in the Neyman series, since the control system is actually a low-frequency filter, we may present the nonlinear functions $y_1$, $y_2$, $y_3$ and $y_4$ in the following way:

$$y_1 = \theta_0^2 \sin\theta_0 [J_0(a) + 2J_2(a) \sin 2U] + \theta_0^2 \cos\theta_0 2J_1(a) \sin U + 2\theta_0 \sin\theta_0 a [J_0(a) + 2J_2(a) \sin 2U] \sin U + 4\theta_0 \cos\theta_0 a J_1(a) \sin^2 U + \sin\theta_0 a^2 [J_0(a) + 2J_2(a) \sin 2U] + 2\cos\theta_0 a^2 J_1(a) \sin^3 U,$$

$$y_2 = [\sin^2\theta_0 a J_0^2(a) \cos U + 2\sin^2\theta_0 a J_0(a) J_2(a)(\sin U + \sin 3U) + \sin^2\theta_0 a J_2^2(a)(2\cos U - \cos 3U - \cos 5U) + \sin 2\theta_0 a J_0(a) J_1(a) \sin 2U + 2\sin 2\theta_0 a J_1(a) J_2(a) \sin^2 2U + \cos^2\theta_0 a J_1^2(a)(\sin 3U - \sin U)]\omega,$$

$$y_3 = -[\sin^2\theta_0 a J_0^2(a) \sin U + 2\sin^2\theta_0 a J_0(a) J_2(a)(\cos U - \cos 3U) + \sin^2\theta_0 a J_2^2(a)(2\sin U + \sin 3U - \sin 5U) + 2\sin 2\theta_0 a J_0(a) J_1(a) \sin^2 U + \sin 2\theta_0 a J_1(a) J_2(a)(2\sin 2U - \sin 4U) + 4\cos^2\theta_0 a J_1^2(a) \sin^3 U]\omega^2,$$

$$y_4 = -[\sin^2\theta_0 a J_0^2(a) \cos U + 2\sin^2\theta_0 a J_0(a) J_2(a)(\sin U + \sin 3U) + \sin^2\theta_0 a J_2^2(a)(2\cos U - \cos 3U - \cos 5U) + \sin 2\theta_0 a J_0(a) J_1(a) \sin 2U + 2\sin 2\theta_0 a J_1(a) J_2(a) \sin^2 2U + \cos^2\theta_0 a J_1^2(a)(\sin 3U - \sin U)]\omega^3$$

With this, the coefficients of the linearized nonlinear functions, defined as

$$q_0^j(a, \theta_0, \omega) = \frac{1}{2\pi}\int_0^{2\pi} y du, \quad q^j(a, \theta_0, \omega) = \frac{1}{\pi a}\int_0^{2\pi} y \sin u du, \quad q_1^j(a, \theta_0, \omega) = \frac{1}{\pi a}\int_0^{2\pi} y \cos u du,$$

have the following expressions:

$$q_0'(a, \theta_0) = \theta_0^2 \sin\theta_0 J_0(a) + 2\theta_0 \cos\theta_0 a J_1(a) + \frac{1}{2}\sin\theta_0 a^2 J_0(a),$$

$$q'(a, \theta_0) = 2\theta_0^2 \cos\theta_0 J_1(a)/a + 2\theta_0 \sin\theta_0 J_0(a) + \frac{3}{2}\cos\theta_0 a J_1(a),$$

$$q_1'(a, \theta_0) = 2\theta_0 \sin\theta_0 J_2(a),$$

$$q_0^{'}(a, \theta_0, \omega) = \sin 2\theta_0\, a\, J_1(a)\, J_2(a)\, \omega,$$

$$q^{''}(a, \theta_0, \omega) = 2\sin^2\theta_0\, J_0(a)\, J_2(a)\, \omega - \cos^2\theta_0\, J_1^2(a)\, \omega,$$

$$q_1^{'}(a, \theta_0, \omega) = \sin^2\theta_0\, J_0^2(a)\, \omega + 2\sin^2\theta_0\, J_2^2(a)\, \omega,$$

$$q_0^{'''}(a, \theta_0, \omega) = -\sin 2\theta_0\, J_0(a)\, J_1(a)\, a\, \omega^2,$$

$$q_2(a, \theta_0, \omega) = -[\sin^2\theta_0\, J_0^2(a)\, \omega^2 + 2\sin^2\theta_0\, J_2^2(a)\, \omega^2 + 2\sin 2\theta_0\, J_0(a)\, J_1(a)\, \omega^2 + 3\cos^2\theta_0\, J_1^2(a)\, \omega^2],$$

$$q_1^{'''}(a, \theta_0, \omega) = -2\sin^2\theta_0\, J_0(a)\, J_2(a)\, \omega^2,$$

$$q_0^{IV}(a, \theta_0, \omega) = -\sin 2\theta_0\, a\, J_1(a)\, J_2(a)\, \omega^3,$$

$$q^{IV}(a, \theta_0, \omega) = \cos^2\theta_0\, J_1^2(a)\, \omega^3 - 2\sin^2\theta_0\, J_0(a)\, J_2(a)\, \omega^2,$$

$$q_2(a, \theta_0, \omega) = -\sin^2\theta_0\, J_0^2(a)\, \omega^3 - 2\sin^2\theta_0\, J_2^2(a)\, \omega^3.$$

The coefficients $q_i^j$ thus found are constant for constant values of a and $\omega$. In making the transition to oscillatory processes with variable a and $\omega$, the values of the coefficients also change.

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# TRANSIENT RESPONSE IN A DC CIRCUIT CONSISTING OF AN OHMIC RESISTANCE AND A THERMISTOR

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For a circuit consisting of a thermistor and an ordinary resistor, the block schematic is given and the transfer function and stability conditions are found. A method is presented for constructing the transient response in this circuit. The investigation is based on a common idealization, the averaging of the thermistor temperature over its conductance.

An electrical circuit consisting of an ohmic resistance and a thermistor reacts not only to variations of the voltage on its terminals and to changes in the magnitude of the ohmic resistance included in the network with it, but also to variations of the thermal relationship of the thermistor with the ambient medium.

Many works have recently appeared which are devoted to the study of the physical properties of thermistors, and to their applications [1-5]. In them, the properties of thermistors are predominantly described and studied in stationary regimens. The few works in which the nonstationary flow of current through thermistors has been investigated considered primarily foreign types of thermistors and, moreover, did not treat the questions fully. The present paper is devoted to the study of a dc $R_T$, R* circuit and to the construction of its transient response. The analysis and computation of the possible states of such an elementary circuit are the first necessary steps in the study of more complicated thermistor circuits. The investigation is based on a common idealization - the averaging of the thermistor temperature over its conductance. The temperature of the thermistor is considered to be a temperature T, uniform for the entire thermistor, the magnitude of which, when substituted in the formula $R_T = R_\infty \exp B/T$, will give the value of the resistance $R_T$ which is actually measured.

## Circuit Equations and Transient Response

Any cause which engenders a change in the conditions of heat transfer between the thermistor and its environment will lead to a redistribution of voltage and power dissipation between the elements R and $R_T$ (Fig. 1). This process continues until the following equations are satisfied:

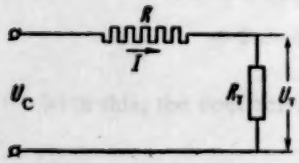


Fig. 1

$$P_{T0} = P_C - P_R,$$

$$P_{T0} = I_0 U_{T0} = \alpha F (T_0 - \theta),$$

$$P_R = I_0^2 R.$$

Here, $I_0$ and $U_{T0}$ are, respectively, the current flowing through the thermistor and the voltage drop across it in the steady state, $\alpha$ is the coefficient of heat transmission of the thermistor with surface area F through the surrounding medium. If there is a stable equilibrium state then, at the termination of the transient response, the

*We will thus denote a circuit consisting of a resistor and thermistor in series.

thermistor is heated to temperature $T_0$. In the process of temperature variation, the power dissipated by the thermistor is partly dissipated to the surrounding medium (power $P_1$) and partly acts to vary the thermal content of the thermistor itself (power $P_2$). Thus,

$$P = P_1 + P_2,$$

where

$$\begin{aligned} P &= I^2 R_T, \\ P_1 &= \alpha F(T - \theta), \\ P_2 &= cV\frac{dT}{dt}, \\ R_T &= R_\infty e^{\frac{B}{T}}, \\ I &= \frac{U_C}{R_T + R}. \end{aligned} \tag{1}$$

Here, c is the thermal capacity and V the volume of the thermistor, and t is the time.

From the system (1) of equations describing the transient response in the circuit, we find that

$$\frac{cV}{\alpha F}\frac{dT}{dt} + T = \theta + \frac{1}{\alpha F}\frac{U_C^2}{R}\frac{R_T/R}{(1 + R_T/R)^2}. \tag{2}$$

It is clear from Eq. (2) that a circuit containing a thermistor is sensitive to changes in any quantity which enters into the right member of the equation. With changes of circuit parameters or in the thermic relationship of the thermistor with its environment, the circuit may become unstable, leading to a rapid increase or decrease in the current flowing through it. In order to derive stability conditions for the circuit, we consider system (1) in the form of linear approximations to deviations from the equilibrium state, introducing the symbol $\Delta$ to denote a deviation:

$$\begin{aligned} \Delta P &= \Delta P_1 + \Delta P_2, \\ \Delta P &= 2U_{T0}\Delta I + I_0^2 \Delta R_T, \\ \Delta P_1 &= \alpha_0 F(\Delta T - \Delta\theta) + F(T_0 - \theta_0)\Delta\alpha, \end{aligned}$$

$$\begin{aligned} \Delta P_2 &= cV\frac{d\Delta T}{dt}, \\ \Delta R_T &= -\beta_0 R_{T0}\Delta T, \\ \Delta I &= \frac{1}{(R_{T0} + R_0)}\Delta U_R - \frac{U_{R0}}{(R_{T0} + R_0)^2}(\Delta R_T + \Delta R). \end{aligned} \tag{3}$$

Here, $\beta_0 = B/T_0^2$.

The quantities $\Delta\theta$, $\Delta\alpha$, $\Delta R$ and $\Delta U_C$ are mutually independent. Therefore, we may take any of them as an input quantity depending functionally on the circuit values. As the output quantity, we may take any one (or several) of the remaining "unknowns," since they are mutually dependent.

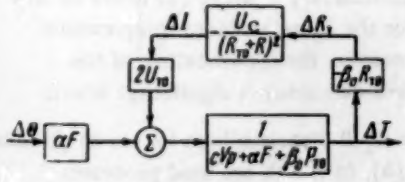

Fig. 2

We now consider two basic cases, the reaction of the $R_T$, R circuit to a change in the temperature of the medium surrounding the thermistor (the input quantity is $\Delta\theta$), and the reaction to a change in the medium or in its state (the input quantity is $\Delta\alpha$).

In both cases, we take as the output quantity the change in temperature of the thermistor. The first case occurs when the thermistor is used as a sensing element of the temperature of a stagnant fluid or gaseous medium. With this, the conditions $U_C$ = = const and R = const usually hold. The coefficient $\alpha$ depends on the difference $T - \theta$. This difference is defined by the accuracy with which the temperature is to be measured, and ordinarily does not exceed a fraction of a degree. We may therefore set $\alpha$ = const [6]. With these conditions holding, we can use Eqs.(3) to set up the block schematic (Fig. 2) and to obtain the circuit's transfer function:

$$\frac{\Delta T(p)}{\Delta\theta(p)} = \frac{K_\theta}{Hp+1},$$

where

$$H = \frac{cV}{\alpha F}\cdot\frac{1}{1-\frac{1}{\alpha F}\beta_\theta\frac{U_{T0}}{U_C}(U_{T0}-U_R)I_\theta}, \quad K_\theta = H\frac{\alpha F}{cV}, \quad p = \frac{d}{dt}. \tag{4}$$

The block schematic, showing the causal chains, was set up by taking into account the deviations from the steady-state value of the temperature of the medium surrounding the thermistor, said variations giving rise to variations in the temperature and resistance of the thermistor. This leads to a corresponding power drain by the thermistor as a function of the current passed, and to a further variation of its temperature. The impact of the feedback loop is determined by the error which can occur due to superheating of the thermistor by the measuring current.

The high accuracy requirements on the measurements determine the position of the working portion on the ascending arm of the thermistor's volt-ampere characteristic (here the difference $T-\theta$ is small, and so Ohm's law is valid). Therefore, the dynamic properties of the circuit being considered can be characterized by the time and amplitude-phase characteristics:

$$\Delta T = K_\theta\Delta\theta\left(1-e^{-\frac{t}{H}}\right), \quad \frac{\Delta T(j\omega)}{\Delta\theta(j\omega)} = \frac{K_\theta}{Hj\omega+1},^{*}$$

where $\omega$ is the oscillatory frequency of the temperature $\theta$.

It is known that with temperature variations of the surrounding medium the volt-ampere characteristic is deformed and, under definite conditions, the $R_T$,R circuit becomes unstable, leading to a rapid increase or decrease of current. It is clear from (4) that the condition of static stability of the circuit has the form:

$$\frac{1}{\alpha F}\beta_\theta\frac{U_{T0}}{U_C}(U_{T0}-U_R)I_\theta < 1. \tag{5}$$

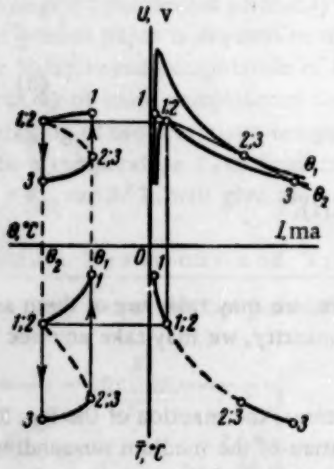

Fig. 3

The points of the volt-ampere characteristic for which $U_{T\theta} < U_R$ or, what amounts to the same thing, $R_{T\theta} < R$, always correspond to stable circuits.

In the first quadrant of Fig. 3 are shown the volt-ampere characteristic of the thermistor for two different ambient temperatures, $\theta_1$ and $\theta_2$, and the lines 1-3 which characterize the ohmic resistance in series with the thermistor. The points labeled with two numbers are the limits of stability. The solid lines in the second, third, and fourth quadrants define the sets of stable points which can be obtained by a continuous variation of the ambient temperature $\theta$ from $\theta_1$ to $\theta_2$. The unstable points lie on the dashed lines. In the temperature interval $\theta_1 < \theta < \theta_2$, the thermistor may be used as a proportional transducer only on the stable segments 1-1,2 and 2,3-3. However, it is inefficient to work on the segment 2,3-3 since the static sensitivity of the thermistor, $\beta_\theta = B/T_\theta^2$, is lower on this segment than on the segment 1-1,2 for the same ambient temperature, and the time constant is greater. Moreover, the superheating of the thermistor, $T-\theta$, on the segment 2,3-3 can reach a significant level.

The presence of unstable equilibrium states (points 1,2 and 2,3) of the $R_T$,R circuit allows its use in circuits for thermal protection of machines and for automatic transformer blow-out [5], in which are used processes leading from point 1,2 to point 3, and conversely (Fig. 3).

*Experimental determinations of the quantity $cV/\alpha F$ and subsequent calculations show that, for example, for a type KMT-1 thermistor placed in transformer oil, with $\omega = 0.01$ second$^{-1}$, the phase shift between $\Delta T$ and $\Delta\theta$ is about 16.5°, and the amplitude deviation is about 4%.

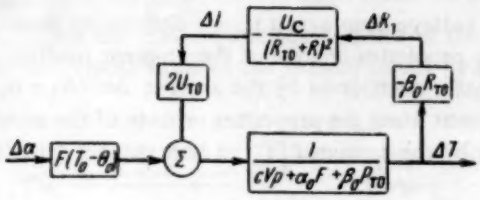

Fig. 4

It is not possible to study these processes in linear approximations, since this gives infinitely large thermistor temperature variations for arbitrarily small increases of ambient temperature above $\theta_2$ or decreases below $\theta_1$. Therefore, we do not limit ourselves to linearity in considering the process of moving from point 1,2 to point 3.

Let the ambient temperature $\theta$ increase and, at a certain moment, become equal to $\theta_2$. Then the imbalance of the powers drained and dissipated by the thermistor, $\Delta P_T = P_T(T) - P_{T_0}(T)$, where $P_T = I(U_C - IR)$ and $P_{T_0} = \alpha F(T - \theta)$, induces a variation in the thermistor temperature, governed by the equation $cV(dT/dt) = \Delta P_T$.

Since there is no analytic expression for $\Delta P_T(T)$, the solution of this equation may be obtained graphically (cf. Appendix 2). For this, it is necessary to know the dependence of $\Delta P_T/cV$ on the temperature, $T = B/\ln[U_C - IR)/IR_{\infty}]$, which is easily constructed from the given static volt-ampere characteristics of the thermistor.*

Let us consider the second case. This case occurs when thermistors are used as indicators (or signallers) of the medium and its state, characterized by the speed, pressure, concentration, etc. Such a use of a thermistor is based on the dependence of the thermistor's heat-transfer coefficient on the ambient medium and its state. As before, Eq. (2) describes the transient response of the circuit when the heat-transfer coefficient varies. In this case, $U_C$ = const, R = const, $\theta$ = const, and the system of equations in (3) may be represented by a block schematic of the causal chains as in Fig. 4. The corresponding transfer function has the form:

$$\frac{\Delta T(p)}{\Delta\alpha(p)} = -\frac{K_\alpha}{Hp+1},$$

where

$$H = \frac{cV}{\alpha_0 F}\cdot\frac{1}{1-\frac{1}{\alpha_0 F}\beta_0\frac{U_{T0}}{U_C}(U_{T0}-U_R)I_0}, \quad K_\alpha = H\frac{F}{cV}(T_0-\theta). \tag{6}$$

We may see, in analyzing Eq. (6), that the working portion of the volt-ampere characteristic must be located at the maximum voltage for which the gain $K_\alpha$ is sufficiently large but the influence of the variations in ambient temperature $\theta$ on the thermistor temperature is small.

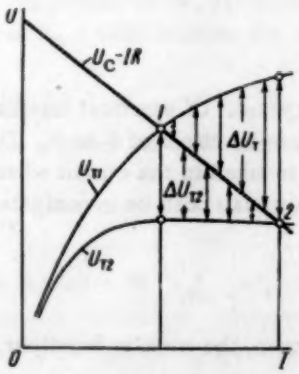

Fig. 5

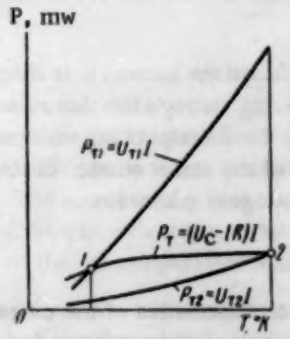

Fig. 6

We shall consider separately the reaction of the $R_T$,R circuit to a step-wise variation in the thermistor's heat-transfer coefficient. This case occurs in level control or medium inspection.

* The method for determining the magnitude of cV is given in Appendix 1.

Let a thermistor be found in a medium which, up to a certain moment, is characterized by the heat-transfer coefficient $\alpha_1$; the current through the thermistor, and the voltage drop across it, are defined by point 1 (Fig. 5). At time t = 0, let there be an instantaneous change in the properties or state of the ambient medium. There will then be a corresponding step-wise jump in the heat-transfer coefficient by the amount $\Delta\alpha = \alpha_1 - \alpha_2$, where $\alpha_2$ is the thermistor's heat-transfer coefficient after the moment when the properties or state of the ambient medium changed. A variation of the thermistor's heat-transfer by the amount $F(T_1 - \theta)\Delta\alpha$ induces a variation of its temperature in accordance with the equation:

$$cV\frac{dT}{dt} = \Delta P_T(T)\ \text{sign}\ \Delta\alpha.$$

Here

$$\text{sign}\ \Delta\alpha = \begin{cases} +1 & \text{for } \Delta\alpha > 0, \\ -1 & \text{for } \Delta\alpha < 0. \end{cases}$$

The instantaneous value of the power consumed by the thermistor in the transient response is determined by the expression $P_T = I(U_C - IR)$. The power dissipated in steady-state heat exchange with the surrounding medium, as characterized by the heat-transfer coefficients $\alpha_1$ and $\alpha_2$, is given by the formulas, respectively, $P_{T1} = \alpha_1 F(T - \theta) = IU_{T1}$, and $P_{T2} = \alpha_2 F(T - \theta) = IU_{T2}$. $|\Delta P_T|$ depends on the sign of $\Delta\alpha$ and is defined by the relationship:

$$|\Delta P_T| = \begin{cases} |P_T - P_{T1}| & \text{for } \Delta\alpha > 0, \\ |P_T - P_{T2}| & \text{for } \Delta\alpha < 0. \end{cases}$$

Figure 6 gives the dependence of $P_T$, $P_{T1}$ and $P_{T2}$ on the temperature.

The magnitude of $P_T$ is given as a function of the instantaneous thermistor temperature during the transient response, $T = B/\ln(R_T/R_\infty)$, where $R_T = (U_C - IR)/I$.

The magnitude of $P_{T1}$ is given as a function of any steady-state temperature of a thermistor immersed in a medium with heat-transfer coefficient $\alpha_1$:

$$T = \frac{B}{\ln\frac{R_{T1}}{R_\infty}} \qquad \left(R_{T1} = \frac{U_{T1}}{I}\right).$$

The magnitude of $P_{T2}$ is a function of the steady-state temperature of a thermistor in a medium with heat-transfer coefficient $\alpha_2$:

$$T = \frac{B}{\ln\frac{R_{T2}}{R_\infty}} \qquad \left(R_{T2} = \frac{U_{T2}}{I}\right).$$

With these functions known, it is simple to construct the transient response. Of practical interest is the fall of current flowing through the thermistor, or the voltage drop across it, as functions of $\theta$ or $\alpha$. These functions are found by the corresponding computations. The transient response induced in the circuit when there is a step-wise jump of the series ohmic resistance, or in the voltage on the terminals, can be investigated and constructed in the analogous manner.

## SUMMARY

1. The block schematics of the causal chains (Figs. 2 and 4) were drawn, the transfer functions, (4) and (6) were obtained, as was the condition for circuit stability, (5).

2. The processes arising in the $R_T$,R circuit given a relay effect, or a step-wise variation in the heat-transfer coefficient, may be considered on the basis of the equation $cV(dT/dt) = \Delta P_T$. The static volt-ampere characteristic and the numerical value of cV are required for the graphic solution of this equation.

3. The block schematics of the causal chains and the transfer functions of the $R_T$,R circuit may be used for its analysis and design.

## Appendix 1

For the analysis and computation of the transient response in a thermistor circuit, it is necessary to know the numerical value of the product of the thermistor's thermal capacity and volume (cV). Given oscillograms of the transient response, for example, from point 1,2 to point 3 (Fig. 7), this quantity may be determined from the equation $cV(dT/dt) = \Delta P_T$. In fact, the right member of this equation can be computed from the given static volt-ampere characteristic, and the left member can be determined from the oscillogram of the process $I = I(t)$, constructed in temperature-time coordinates (Fig. 8). This curve has a point of inflection which corresponds to the maximum value of $\Delta P_T = \Delta P_{T\,max}$. To determine the magnitude of cV, it is convenient to use the linear segment of the curve in the neighborhood of the point where dT and dt can be replaced by, respectively, $\Delta T$ and $\Delta t$.

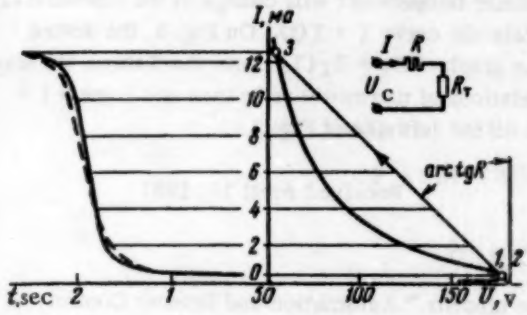


Fig. 7

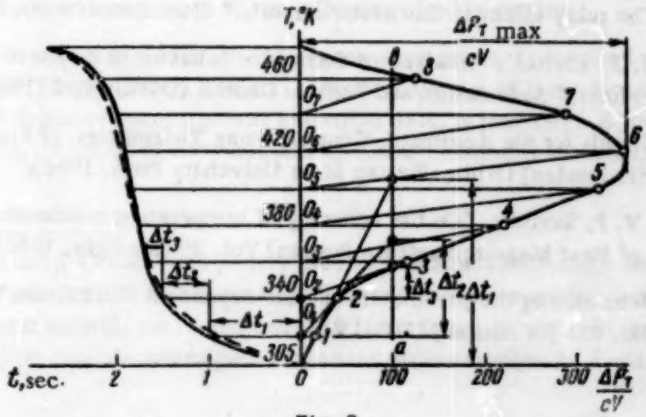


Fig. 8

Consequently, $cV = \Delta t \Delta P_{T\,max} / \Delta T$.

The magnitude of cV, determined in the foregoing manner for a type KMT-11 thermistor with a nominal resistance of $R_{20} = 1690$ kilohm, $B = 4600°K$, $R_{\infty} = 0.281$ ohm, was equal to 1.4 milliwatt second/degree.

## Appendix 2

Below is given the construction of the process of current increase in a circuit containing a type KMT-11 thermistor immersed in transformer oil at a temperature of 20.7°C and a linear resistance of R = 10 kilohm. The increase of current engendered a disruption of the circuit's static stability due to the increase of the oil temperature above 20.7°C.

TABLE

| I, ma | $R_T$, kohm | T, °K | $P_T$, mw | $\Delta P_T$, mw |
|---|---|---|---|---|
| 0.2 | 1047.5 | 305.0 | 41.8 | 0 |
| 0.3 | 694.2 | 313.0 | 62.5 | 9.5 |
| 0.4 | 517.5 | 319.0 | 82.8 | 12.0 |
| 0.5 | 411.5 | 324.0 | 102.8 | 15.9 |
| 1.0 | 199.5 | 342.0 | 199.5 | 41.5 |
| 2.0 | 93.5 | 362.0 | 374.0 | 128.0 |
| 4.0 | 40.5 | 388.0 | 648.0 | 254.0 |
| 6.0 | 22.8 | 407.0 | 822.0 | 348.0 |
| 8.0 | 14.0 | 427.0 | 896.0 | 336.0 |
| 10.0 | 8.7 | 446.0 | 870.0 | 250.0 |
| 12.0 | 5.2 | 468.0 | 748.0 | 60.0 |
| 12.5 | 4.5 | 476.0 | | 0 |

To obtain the transient process from point 1,2 to point 3 of the thermistor's static volt-ampere characteristic (Fig. 7), we construct the graph (the right portion of Fig. 8) of $\Delta P_T / cV$ as a function of the instantaneous thermistor temperature, calculated from the formula $T = B/\ln(R_T/R_{\infty})$, where $R_T = (U_C - IR)/I$, and $U_C = 180$ volts. The data necessary for the construction of the graph of $\Delta P_T/cV = f(T)$

are given in the Table. One then draws the line ab, parallel to the axis of ordinates, at some distance which is convenient for the construction (in the given case, equal to 100) and n equidistant lines [$n = (T_3 - T_{1,2})/\Delta T$, $\Delta T = 20°$] parallel to the axis of abscissas. The distances between the points of intersection of the lines 0-1, $0_1$-2, . . . , $0_{n-1}$-n with the line ab and the corresponding equidistant lines give the time increments $\Delta t_1$, $\Delta t_2$, . . . , $\Delta t_n$. During each of these increments of time, the thermistor temperature will change by the amount $\Delta T$. Measuring off the $\Delta t$ to the left of the axis of ordinates, we obtain the curve $T = T(t)$. On Fig. 8, the dotted line gives the oscillogram of the process $T = T(t)$. Knowing the graph of $R_T = R_T(T)$ (from the Table), we may construct the curves showing the dependence on time of the variations of thermistor resistance and current $I = U_c/[R_T(t) + R]$. This latter curve is given by the dotted line on the left side of Fig. 7.

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* See C.B. Translation.

# TRANSIENT RESPONSES IN THE MAGNETIC CIRCUITS OF ELECTROMAGNETIC CLUTCHES

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(Moscow)

A method is proposed for the engineering calculations of transient responses in the magnetic circuits of electromagnetic clutches consisting of several solid members, account being taken of the influence of eddy currents.

The equations for the transient responses are given, as are the formulas to be used in the computations, and an example of the use of these formulas, together with a comparison of computed and empirical data, is provided. A more precise exposition of the influence of eddy currents on rotor motion is included.

## 1. Posing of the Problem

Electromagnetic clutches (EMC) are widely used in machine construction and in automatic devices. In the majority of cases in which EMC are used – in high-speed executive mechanisms, electromagnetic brakes, relay systems of automatic control, etc. – the essential factor is the behavior of the clutch during transient responses determined by its own electromagnetic transient responses when its excitation circuit is switched on and off.

The majority of clutch designs have solid magnetic circuits.* In such clutches the eddy currents have decisive importance in determining the course of the transient response when the circuit is switched off and, to a lesser extent, when it is switched on. The influence of the eddy currents is particularly significant during switching off when, as is generally the case, the external circuit is open and the course of the transient response is completely determined by the parameters of the magnetic circuit. However, the existing methods for calculating electromagnetic clutches [1-4] do not take into account the effect of eddy currents and, in practice, do not allow computation of the transient response when the clutch is released and, moreover, lead to errors in computing the effect of forced switching-in. The absence of a theoretical solution and of a method for engineering calculations, taking into account the inertia of a clutch considered as an electromagnetic system with solid magnetic circuits, renders difficult both the choice of clutch parameters in the design phase and the analysis of control systems which contain EMCs.

The present paper presents the fundamental results of a theoretical and experimental investigation into the transient responses in electromagnetic clutches with the effect of eddy currents taken into account. The problems to be considered are the finding of general equations for the electromagnetic transient responses and the development of methods for their engineering computation.

The investigation was carried out so as to be applicable to single-disk electromagnetic friction clutches. We have also considered the special features involved in computing clutches with rotor normally switched in, as well as powdered-metal and magneto-emulsion clutches. The method of investigation and our results may be used for solving the analogous problems for other types of electromagnetic clutches, as well as for various electromagnetic systems with small armature travels (for example, relays).

*We have excluded certain types of clutches where the executive organ employs powdered metal or emulsions.

## 2. The Method

Of prime importance in the problem under consideration is the analytic expression for the effect of eddy currents on the course of the transient response. To obtain such expressions for magnetic circuits consisting of several solid members, one might use the classical methods of field theory,* of equivalent short-circuited windings [5], or reluctance operators [6].

We will make use here of the latter method, thus allowing ourselves to employ the well-known advantages of the operator calculus.

This method was previously employed in three cases of which we know for the determination of the inertia of solid cores [6-8].

In the works cited there was considered the relatively simple question of the effect of one solid member of the magnetic circuit on the transient response of the system. Therefore, their manner of employing the general method [6] and, in particular, their method for engineering calculations, was applied by these authors only to particular cases and cannot be used by us in view of the complex configurations of the magnetic circuits of EMCs and the increased requirements as to simplicity and accuracy. In our case it is necessary to find a more general and, at the same time, simpler method of carrying out the calculations.

We note, in accordance with the method chosen, the basic statements which determine the sequence of the steps in the solution.

1) The magnetic circuit is rationally decomposed into a series of members, for each of which there is defined a magnetic admittance operator $G_i(p)$ and a reluctance operator

$$R_i(p) = \frac{1}{G_i(p)} . \tag{1}$$

2) The total reluctance operator is computed

$$R(p) = \sum_{i=1}^{i=k} R_i(p) \tag{2}$$

as well as the resulting induction operator

$$L(p) = \frac{W^2 \cdot 10^{-8}}{R(p)} . \tag{3}$$

3) Using the known L(p) and R(p), we find the ampere-turns operator Wi(p) from the equation

$$i(p) = \frac{U(p)}{r_k + pL(p)} \tag{4}$$

and the operator for magnetic flux on switching in, $\Phi(p)$:

$$\Phi(p) = \frac{Wi(p)}{R(p)} . \tag{5}$$

We now make the transition to the original functions, i.e., to functions of time.

4) The analogous problem for switching-off of the clutch is solved.

5) An analysis of the fundamental equations of the processes is made, and a whole set of questions related to the engineering computations of the transient responses is considered.

Assumptions. Naturally, the analytic solution of the problem posed cannot be carried out with all the actual physical conditions taken into account. We therefore make the following assumptions: 1) the magnetic circuit is not saturated, i.e., $\mu$ = const; 2) flux leakage is small; 3) the induction vector $\overline{B}$ for each portion of

*While it is theoretically possible to solve a system of field equations in partial derivatives, it is tremendously difficult in practice.

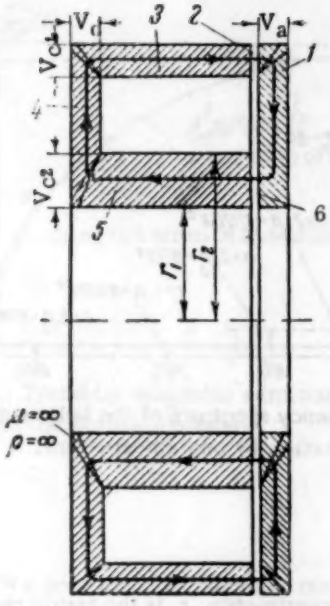


Fig. 1. Sketch of the clutch's magnetic circuit.

the magnetic circuit is parallel to the bounding surface of the portion; and 4) in the gaps between the solid portions, i.e., in the zones in which the vector $\overline{B}$ changes its orientation, the magnetic circuit is ideal, i.e., $\mu = \infty$, $\rho = \infty$.

The first two assumptions are usually made, and are sufficiently well satisfied for single-disk clutches. In the case of parallel bounding surfaces, the third assumption coincides with the second. The fourth assumption is valid if the reluctance of the orientation zones is small in comparison with the reluctance of the magnetic circuit where, of course, the air gaps are neglected, and this condition is always satisfied.

It should also be mentioned that, subsequently, after the basic relationships are obtained, the first two assumptions may be dropped, and the corresponding factors be taken into account.

Turning directly now to the solution of the problem, we decompose the clutch's magnetic circuit into the following portions (Fig. 1): 1 is the rotor disk, 2 is an air gap, 3 is the outside cylindrical portion, 4 is the faceplate disk of the magnetic circuit, 5 is the inside cylindrical portion, and 6 is again an air gap.

Leaving aside the air gaps, we find four portions making up the clutch's magnetic circuit, portions which are pairwise identical in form – two disks with apertures in their centers and two hollow cylinders.

The next step is to determine the parameters of the clutch's magnetic circuit in the transient response – the reluctance operator R(p) and the induction operator L(p).

## 3. Parameters of the Clutch's Magnetic Circuit in the Transient Response

For the determination of R(p) and L(p), in accordance with (2) and (3), it is necessary to compute the reluctance (or admittance) operator of a disk and a hollow cylinder.

By solving Maxwell's equations for a portion, assuming an excitation of one ampere-turn [6], we obtain the transient magnetic admittance of the disk, for a radial flow of flux, in the form

$$G_{\mathrm{d}}(p) = \frac{1}{R_{\mu\mathrm{d}}} \sum_{n=1}^{\infty} \frac{\beta_{n\mathrm{d}}}{1 + pT_{\mu n\mathrm{d}}}, \quad (n = 1, 3, 5, \ldots),$$
$$\beta_n = \frac{8}{n^2\pi^2}, \quad T_{\mu n} = \frac{1}{k\lambda_n}, \quad k = \frac{10^9\rho}{0{,}4\pi\mu}, \quad \lambda_n = \frac{n^2\pi^2}{b^2}. \tag{6}$$

Figure 2 shows the frequency spectrum of the transient magnetic admittance of the disk. The transient magnetic admittance of a hollow cylinder for a flux directed parallel to a generator has the form:

$$G_{\mathrm{c}}(p) = \frac{1}{R_{\mu\mathrm{c}}} \sum_{n=1}^{\infty} \frac{\beta_{n\mathrm{c}}}{1 + pT_{\mu n\mathrm{c}}} \tag{7}$$

Formula (7) was obtained by solving Maxwell's equations and then employing an asymptotic expression for the Bessel functions in the computations for $\beta_{n\mathrm{c}}$ and $\lambda_n$. . . . . .*

$$\beta_n = \frac{4}{n^2\pi^2} \frac{(r_2 - r_1)\left[\sqrt{r_2} - (-1)^n\sqrt{r_1}\right]}{(r_1 + r_2)\left[\sqrt{r_2} + (-1)^n\sqrt{r_1}\right]},$$

where n = 1,2,3,4, . . . .

*Omission in original Russian

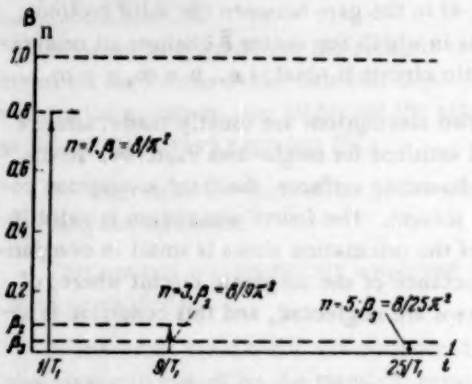


Fig. 2. Frequency spectrum of the disk's field for excitation by one ampere-turn.

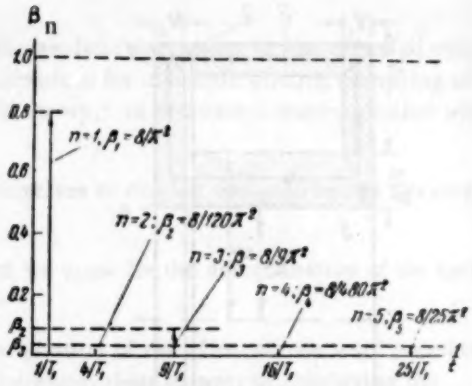


Fig. 3. Frequency spectrum of the hollow cylinder's field for excitation by one ampere-turn.

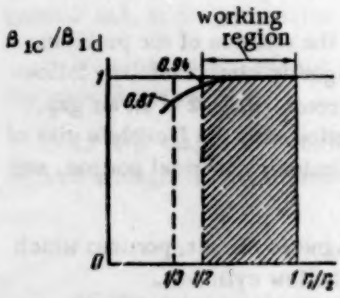


Fig. 4. Dependence of $\beta_{1c}/\beta_{1d}$ on $r_1/r_2$.

$r_2$ is the outside radius of the cylinder in centimeters, $r_1$ is the inside radius of the cylinder in centimeters, and

$$T_{\mu n} = \frac{1}{k\lambda_n}, \quad \lambda_n \approx \frac{n^2\pi^2}{(r_2 - r_1)^2}\left(\frac{r_2}{r_1 + r_2}\right)^2.$$

Figure 3 shows the frequency spectrum of $G_c(p)$.

It follows from Fig. 4 that, for $r_1/r_2 \geq 1/2$ we may, with an error not greater than 6%, take

$$\beta_{nc} = \beta_{nd} = \frac{8}{n^2\pi^2}, \tag{8}$$

where n = 1, 3, 5, . . .

Thus, expressions (6) and (7) are identical. Their original function in the time domain is given on Fig. 5. With the identity n = 2k − 1, n = 1, 3, 5, . . . , k = 1, 2, 3, . . . , and with the use of the relationship (from page 50 of [10])

$$\sum_{k=1}^{\infty} \frac{1}{(2k-1)^2 + x^2} = \frac{\pi}{4x} \operatorname{th} \frac{x\pi}{2}, \tag{9}$$

we may write expressions (6) and (7) in closed form

$$G(p) = \frac{1}{R_\mu} \frac{\operatorname{th} z}{z}, \tag{10}$$

where $z = \pi\sqrt{pT_1}/2$, and $T_1 = T_{\mu n}$ for n = 1.

By setting $\tanh z / z = 1/f(p)$, we obtain the following expression for the disk's reluctance operator

$$R_d(p) = R_{\mu d} f_d(p). \tag{11}$$

For the hollow cylinder we find, analogously,

$$R_c(p) = R_{\mu c} f_c(p). \tag{12}$$

We now sum the reluctance operators of all k portions of the clutch's magnetic circuit (k is the number of portions having different time constants $T_{\mu n}$):

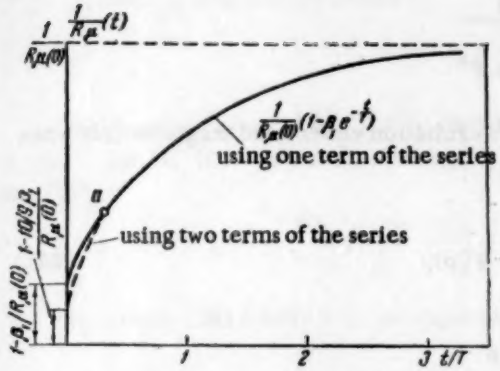


Fig. 5. Transient magnetic admittance of the disk and the hollow cylinder.

$$R(p) = \sum_{i=1}^{k} R_{\mu i} f_i(p). \qquad (13)$$

Going now to relative quantities, we write the clutch's reluctance operator in the form

$$R(p) = R(0) \sum_{i=1}^{k} \alpha_i f_i(p). \qquad (14)$$

In expression (14)

$$R(0) = \sum_{1}^{k} R_{\mu i}, \qquad (15)$$

$$\alpha_i = \frac{R_{\mu i}}{\sum_{1}^{k} R_{\mu i}}. \qquad (16)$$

We note that, in magnetic circuits similar to the one under consideration, it is reasonable to assume that the phenomenon of flux displacement occurs also in the air gaps [7]. In this case, the reluctance of the air gap is taken into account by introducing an equivalent magnetic permeability.

$$\mu_e = \mu \frac{\sum_{1}^{k} R_{\mu i}}{\sum_{1}^{k} R_{\mu i} + R_\delta}. \qquad (17)$$

Thus, the reluctances $R_{\mu i}$ and $R(0)$, as well as the time constant $T_{\mu n}$, contain the equivalent magnetic permeability $\mu_e$.

Having (14) we may easily write the expression for the clutch's inductance operator:

$$L(p) = L(0) \frac{1}{\sum_{1}^{k} \alpha_i f_i(p)}. \qquad (18)$$

Knowing (14) and (18), we may now turn to the determination of the laws governing the rise and fall of the magnetic flux in the EMC.

## 4. Establishing the Flux when the Clutch is Switched On and Off

We now write the equilibrium equations for the clutch's electric and magnetic circuits:

$$U(p) = i(p) r + W p \Phi(p) 10^{-8}, \qquad (19)$$

$$\Phi(p) = \frac{W i(p)}{R(p)}. \qquad (20)$$

By solving (19) and (20) simultaneously, we obtain

$$i(p) = I_0 \frac{1}{1 + pT(p)}, \qquad (21)$$

$$\Phi(p) = \Phi_0 \frac{1}{\sum_{1}^{k} \alpha_i f_i(p) + pT_0}, \qquad (22)$$

where $U(p) = U = \text{const}$, $I_0 = U/r$, $\Phi_0 = \Phi(0)$, and

$$T(p) = \frac{L(p)}{r} = T_0 \frac{1}{\sum_1^k \alpha_i f_i(p)}.$$

Expressions (21) and (22) describe the increase of the clutch's excitation current and magnetic flux when the clutch is switched in by a voltage U = const with a.. air gap $\delta$.

From (22) we find the law of flux decrease:

$$\Phi_{off}(p) = \Phi_{steady} - \Phi(p), \tag{23}$$

where

$$\Phi_{steady} = \Phi_0,\ T_0 = 0.$$

We substitute (22) in (23) obtaining, after some transformations,

$$\Phi_{off}(p) = \Phi_0\left[1 - \frac{1}{\sum_1^k \alpha_i f_i(p)}\right]. \tag{24}$$

We emphasize that the values of $\Phi_0$ and $T_{\mu n}$ which enter into $f_i(p)$ are chosen for the permeability $\mu_e$ corresponding to the given gap $\delta$.

Expressions (22) and (24) describe the transient response with respect to flux in a general form. Below we carry out their transformation to forms convenient for engineering calculations.

## 5. Armature Motion

The transient response when a friction EMC is switched on may be decomposed into three intervals: the starting interval, in which the gap is $\delta = \delta_0 = \text{const}$; the interval of motion, in which the gap $\delta$ is variable; and the interval of steady-state flux (with a tightened rotor), in which the gap is $\delta = \delta_{min} = \text{const}$.

The relationships obtained above were derived for $\delta = \text{const}$, i.e., for an immovable rotor. The dynamics of rotor motion turn out to have a definite effect on the course of the transient response. This effect appears both within the interval of motion, due to the appearance of an additional counter-emf, and outside of it, since the course of the transient response within the interval of motion determines the boundary conditions for the steady-state interval which follows it.

A host of methods for solving the equations of motion of the rotor (armature) of a piece of electromagnetic apparatus are known [11-14]. Of these, as was correctly stated in [1], the most useful is the solution given in [13] which describes the motion of an electromagnet's armature. However, the unfounded assumption was made in [1] that the starting current $i_{st} = 0$. This is not in correspondence with the physics of the phenomena, and leads to a noticeable error in the computations for even small counteracting forces. As shown by analysis, two cases must be distinguished in considering rotor motion in a clutch: a) $\Phi_{st} \ll \Phi_0$, and b) $\Phi_{st} < \Phi_0$.

In case a) the solution in [1] is correct if, instead of U, the expression used is*

$$U_{st} = U - i_{st}r. \tag{25}$$

In case b) the equation of rotor motion becomes linear. In fact,

$$m\frac{d^2x}{dt^2} = A[\Phi_{st} + \Phi_1]^2 - A\Phi_{st}^2, \tag{26}$$

where $x$ is the anchor travel, $\Phi_{st} + \Phi_1 = \Phi$ is the magnetic flux, $\Phi_{st}$ is the starting flux, $\Phi_1 = \Phi - \Phi_{st}$ is the "excess" ("dynamic") flux, and

$$A = 2/(25 \cdot 10^6 S_{pole}).$$

*Here, and in what follows, $i_{st}$ is the starting current computed with eddy currents taken into account.

By transforming (26) we obtain

$$m\frac{d^2x}{dt^2} = A\Phi_1[2\Phi_{st} + \Phi_1]. \tag{27}$$

The greatest value of $\Phi_1 = \Delta\Phi$ is reached at the termination of rotor motion. The mean value is $\Phi_{1me} \approx \Delta\Phi/2$. It is obvious that for $\Phi_{st} > \Delta\Phi$ it is possible to neglect the second term, and to write (27) in the form

$$m\frac{d^2x}{dt^2} = 2A\Phi_{st}\Phi_1. \tag{28}$$

By solving (28) jointly with the equation

$$U_{st} \approx W\frac{d\Phi_1}{dt}10^{-8},$$

we obtain, as an approximate solution, the function

$$x = n't^3, \tag{29}$$

where n' is a constant coefficient which depends on the clutch parameters and t is time.

The increase in flux from the value $\Phi_{st}$ is described by the equation

$$\Phi_1 = \frac{U_{st}}{W}\cdot 10^8 t. \tag{30}$$

Line (30) is tangent to the flux curve at the moment of starting, i.e., at the point $t = t_{st}$. Thus, for the construction (or computation) of this line, it suffices to know the time of motion

$$t_{mo} = \sqrt[3]{\frac{\delta_{max} - \delta_{min}}{n'}}. \tag{31}$$

Thus, in the first approximation, the effect of eddy currents on the process of rotor motion reduces to a decrease of the acting voltage $U_{st}$ at the cost of an absolute and a relative increase in $i_{st}$. This effect leads to a decrease in the speed of flux increase and to an increase in the time of rotor motion.

Having (29), we may write $\delta(t)$, $\mu_e(t)$ and $i(t)$ inside the interval of motion, and then seek a solution for the motion to a second approximation. In practice, the increase in refinement thus obtained is nonessential.

## 6. Engineering Computations of the Transient Response

The laws of magnetic flux increase (22) and decrease (24) were presented in operator form, as derived above. A direct transition from the p-domain to the time domain is possible, but not convenient here, since the complexity of the time functions thus obtained proscribes their use for practical calculations.

The methods, described in the literature, of obtaining approximate solutions in analogous problems [6-8, 15], come to the following.

A. The first terms in the series expansions for $G_i(p)$ are used [6,8].

B. The actual cross section of the solid member is replaced by an equivalent cross section whose shape permits a summation of the series for $G_i(p)$ [7].

C. Graphicoanalytic solutions are employed [15].

The disadvantage of method A is the sizeable methodical error at the beginning of the process (19%). Method B is useful only for the special case of a rectangular core cross section (we note that in our case, the series for $G_i(p)$ are added directly for the actual cross sections). Method C requires the construction of the graph of the denominator of (24) for the operator function, and is therefore extremely arduous. Methods A and B are also sufficiently difficult. Thus, none of the methods enumerated satisfies the requirements for an engineering solution of the problem under consideration.

Here, various variants of an engineering solution will be obtained by the use of approximate values of the "exact" sums:

$$f_i(p) = z_i \coth z_i. \tag{32}$$

Variation 1. If the exact solution at the beginning of the process is nonessential, method A should be used, i.e., a rapidly converging series, of the form given in (6), is to be used, bearing in mind that

$$z_i \operatorname{cth} z_i = \frac{1}{\sum_{n=1}^{\infty} \frac{\beta_n}{1+pT_{in}}} \approx \frac{1+pT_i}{1+(1-\beta)\,pT_i} \qquad (n = 1,\ 3,\ 5,\ \ldots). \tag{33}$$

Substitution of (33) in (22) and (24) leads to the rational-fraction functions

$$\Phi(p) = \frac{b_0 p^k + b_1 p^{k-1} + \cdots + b_{k-1} p + b_k}{a_0 p^{k+1} + a_1 p^k + \cdots a_k p + a_{k+1}}, \tag{34}$$

$$\Phi_{\text{off}}(p) = \frac{(m_0 - b_0)\,p^k + (m_1 - b_1)\,p^{k-1} + \cdots + (m_{k-1} - b_{k-1})\,p}{m_0 p^k + m_1 p^{k-1} + \cdots + m_{k-1} p + m_k}. \tag{35}$$

When we make the transition to the time domain for the cases of switching on, (34), and switching off (35), we obtain, respectively,

$$\Phi(t) = \Phi_0 \left[1 + \sum_{i=1}^{k} A_i e^{-p_i t}\right], \tag{36}$$

where the $p_i$ are roots of the denominator of (34), $\sum_1^k A_i = -1$, since $\Phi(0) = 0$, $1 + \sum_{i=1}^{k} A_i = 0$ and

$$\Phi_{\text{off}}(t) = \Phi_0 \sum_1^k B_i e^{-p_i t}, \tag{37}$$

where the $p_i$ here are the roots of the denominator of (35), $\sum_1^k B_i = \beta = \frac{8}{\pi^2} = 0.81$, since, from (35), $\Phi_{\text{off}}(0) = [(m_0 - b_0)/m_0]\Phi_0 = \beta\,\Phi_0$.

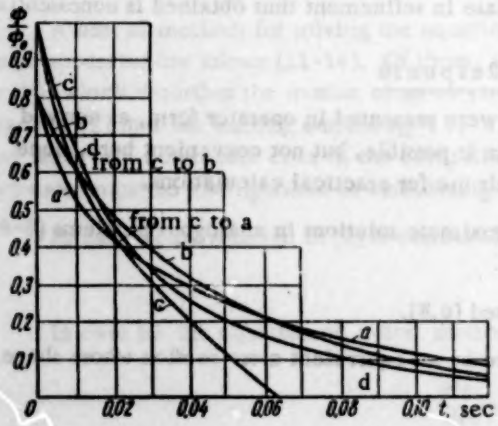


Fig. 6. Variations of the engineering calculations of the transient responses with the clutch switched off for $\delta_{\min} = 0.35$ millimeters.

The expressions for the coefficients $a_i$, $b_i$ and $m_i$ are given in Appendix I, while the calculated curve of $\Phi_{\text{off}}(t)/\Phi_0$, constructed in accordance with (37), is shown in Fig. 6 (curve a).

Even though k is ordinarily 2 to 4, the solution of the k-th degree equation, which is necessary to determine the k roots of the denominator of (37), requires a definite expenditure of time. In this sense, there is a significant amount of convenience in a variation of the approximate solution which is best called a linear averaging of the process.

Variation 2. For a small (principal) root $p_1$ of the denominator of (35), we neglect the influence of the rapidly damped components deriving from the coefficient $(1 - \beta)$, and we set the terms in (35) which contain this coefficient equal to zero. Then, the denominator of (35) is linearized, and its sole root turns out to equal

$$p_1 = -\frac{1}{m_{k-1}}, \tag{38}$$

where

$$m_{k-1} = \sum_{1}^{k} \alpha_i T_i.$$

We then introduce the time constant

$$T_e = \sum_{i}^{k} \alpha_i T_i. \tag{39}$$

This time constant can be defined as a certain equivalent exponent which describes the process of magnetic flux decrease:

$$\Phi_{off}(t) = \beta \Phi_0 e^{-\frac{t}{T_e}}. \tag{40}$$

The curve for the process, computed in accordance with (40), is shown in Fig. 6 (curve b). Physically, the linear averaging is equivalent to the assumption that the solid portions have been replaced, for the basic portions (81%) of the flux, by short-circuited windings with no inherent leakage.

Mathematically, it is a variant of the well-known method of approximate calculation of the small roots of a polynomial.

It follows from a comparison of curves a and b of Fig. 6 that the elementary and simple solution by (40) entails almost no increase in error, so that such a solution is very convenient for rapid estimates in the design of an EMC.

For the process of switching on, the linear averaging gives:

$$\Phi(p) = (1-\beta)\Phi_0 \frac{1}{1+pT_0} + \beta\Phi_0 \frac{1}{1+p(T_e+T_0)}. \tag{41}$$

The time function corresponding to (41) will be

$$\Phi(t) = (1-\beta)\Phi_0(1-e^{-\frac{t}{T_0}}) + \beta\Phi_0(1-e^{-\frac{t}{T_0+T_e}}). \tag{42}$$

In the limiting cases we have:

a) for $T_e = 0$ (no eddy currents)

$$\Phi(t) = \Phi_0(1-e^{-\frac{t}{T_0}}); \tag{43}$$

b) for $T_0 = 0$, $T_e \neq 0$ (supply current)

$$\Phi(t) = (1-\beta)\Phi_0 + \beta\Phi_0(1-e^{-\frac{t}{T_e}}) \tag{44}$$

It is possible to bring (42) to the form

$$\Phi(t) = \Phi_0(1-e^{-\frac{t}{T_0}}) - \beta\Phi_0(e^{-\frac{t}{T_0+T_e}} - e^{-\frac{t}{T_0}}), \tag{42a}$$

where the second term describes the effect of the eddy currents upon switching-on. The degree of this effect is estimated by the ratio $T_e/T_0$.

Variation 3. Relationships (37) and (40) have an increased error (up to 19%) for small t. For ordinary friction clutches this fact has little practical effect on the computed release time. However, for powder clutches, by virtue of the discontinuity of the flux functions, and for clutches with normally switched-in rotors, possessing significant counteracting forces, it is necessary to increase the accuracy of the computations for small t (for large p). An approximate value of the sum $1/f_1(p)$ is given, for large p, by the equality

$$z_i \coth z_i \approx z_i, \tag{45}$$

where

$$z_i = \frac{\pi \sqrt{pT_i}}{2}.$$

Substitution of (45) in (24) gives

$$\Phi^*_{off}(p) = \Phi_0\left[1 - \frac{1}{\tau_e \sqrt{p}}\right], \tag{46}$$

where

$$\tau_e = \frac{\pi}{2}\sum_1^k \alpha_i \sqrt{T_i}. \tag{47}$$

Going over to the corresponding time function, we write

$$\Phi^*_{off}(t) = \Phi_0\left[1 - \frac{2}{\sqrt{\pi}}\frac{\sqrt{t}}{\tau_e}\right]. \tag{48}$$

At switching on, if $T_0 \ll T_e$ ("supply current"), the process is described by the equation

$$\Phi^*(t) = \frac{2}{\sqrt{\pi}}\frac{\Phi_0}{\tau_e}\sqrt{t}. \tag{49}$$

It should be borne in mind the formulas (48) and (49) are valuable only for small $t$; in practice, the limits of their applicability occur at(1 to 0.5)$\Phi_0$ for falling flux and (0 to 0.5)$\Phi_0$ for rising flux. The computed curve constructed in accordance with (48) is shown on Fig. 6c.

It is obvious that the computation of the process of flux fall will be complete if, on one graph, the curves for (48)and (40), or (48) and (37) are drawn. The transition from the functional dependence of (48) to that of (40), or of (37), occurs at the point at which the curves intersect, a point which always exists.

For the switching-in process when $T_0 \ll T_e$ (forcing by a large additional impedance $R_{add} \gg r_k$), Formulas (49) and (44) should be used and, for commensurate $T_0$ and $T_e$, Formula (42). When there is no forcing, ordinarily $T_0 \gg T_e$, and the eddy currents can be neglected in computing the processes occurring at switching-in.

## 7. Effects of the Air Gaps and of Magnetic Circuit Saturation

Under our assumptions, both these factors affect only the magnitude of $\mu_e$ and it is therefore convenient to consider them simultaneously. Due either to variations in the air gap or to nonlinearities in the magnetization curve, the equivalent permeability will be assumed to take a new value:

$$\mu'_e = \eta\,\mu_e \quad (\eta \neq 1). \tag{50}$$

Then, all the time constants will also change by a factor of $\eta$:

$$T'_i = \eta\, T_i, \tag{51}$$

$$T'_0 = \eta\, T_0. \tag{52}$$

For (35), for example, we then obtain

$$\Phi_{off}(p) = \Phi_0\,\frac{(m_0 - b_0)\,\eta^k p^k + (m_1 - b_1)\,\eta^{k-1}p^{k-1} + \cdots + (m_{k-1} - b_{k-1})\,\eta p}{m_0\eta^k p^k + m_1\eta^{k-1}p^{k-1} + \cdots + m_{k-1}\eta p + 1}. \tag{53}$$

By introducing the new operator

$$q = \eta p, \tag{54}$$

we obtain a function which is literally identical to (35). The roots of its denominator will be $q_i = p_i$, whence the new attenuation values will be

$$p'_i = \frac{1}{\eta}\,p_i. \tag{55}$$

The values of the numerator obviously do not change. Analogous results are obtained by the substitution of $\mu'_e = \eta\,\mu_e$ in Formulas (34), (48), and others.

We thus arrive at the following rule: for a given EMC, the form of the law according to which the magnetic flux is established does not depend, for a motionless rotor on the magnitude of the air gap or on the magnetic permeability. The effect of these factors reduces to a variation in the scale of the times for the transient response curves, a scale change proportional to the relative variation of the resulting magnetic admittance of the magnetic circuit.

The rule just given makes it easy to find the magnitudes of the gaps $\delta_{min}$ and $\delta_{max}$ from the conditions on the required release of braking times and, for powder clutches, to obtain given dynamic characteristics.

The conclusion may be drawn from what has been here presented that the effect of the magnitude of $\mu$ on the process is small. When it is necessary to take this effect more accurately into account, a piecewise linear magnetization characteristic (in two pieces) should be used, in conjunction with the well-known rules of curve fitting.

## 8. Taking Leakage into Account

The phenomenon of leakage corresponds to the appearance of a parallel magnetic admittance $G_s$ which can be introduced into expressions (14) and (18) and be thus taken into account analytically.

The admittance $G_s$ is inertialess with respect to the eddy currents and somewhat decreases their effect. On the other hand, the additional component of the magnetic flux, corresponding to the leakage admittance, leads to a certain decrease of the magnetic potential at the boundaries of the air gap and, consequently, somewhat retards the increase of flux. As a result of the interaction of these mutually opposed factors, the effect of leakage on the transient response is small. A more detailed investigation shows that when there are eddy currents, the leakage coefficient varies during the transient response and is an additive factor to the transient impedance.

Due to the small effect of leakage on the transient response, the introduction of a leakage coefficient operator into the computational formulas is not worthwhile, since this would significantly complicate the computations without leading to any essential change in the final results.

Thus, there should be used in the computations only the ordinary "stationary" leakage coefficient, depending on rotor position, and the steady-state value of the magnetic flux, leakage being taken into account, should be substituted into the computational formulas.

The error arising from this, as with the error due to approximate treatment of saturation, enters into the total error of the computations.

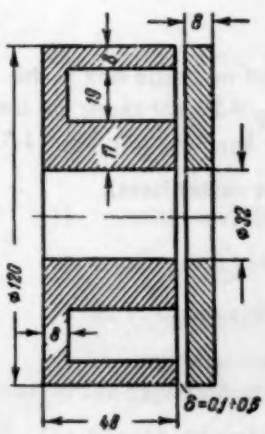


Fig. 7. Sketch of the clutch's magnetic circuit.

## 9. Experimental Data and Computational Error

An experimental verification of the computational results* was performed on the electromagnetic single-disk clutch (Fig. 7), the magnetic circuit of which was implemented by the type of magnetic circuit in the electromagnetic clutch ENIMS, series EM (leading material A-100-1, 1955). An oscillogram of the transient response of the flux itself (Fig. 6, curve d) was obtained by including a testing winding, with a number of turns $W_t = 30$, in series with an integrating RC circuit leading to the dc amplifier at the output of the oscillograph loop. It follows from a comparison of the oscillogram with the computed curves that the maximum error of computation does not exceed 20% to 25%.

## SUMMARY

1. We obtained general equations for the transient responses in the magnetic circuits of electromagnetic clutches, taking into account the inertia of the solid magnetic circuits and the excitation circuits of the clutches.

*The experimental work was carried out in the electrical branch of the Experimental Research Institute for Metal-Cutting Machines.

2. We developed an engineering method for computing the transient responses, allowing compact computational formulas to be obtained.

3. The computations made in accordance with the formulas presented are carried out comparatively quickly and give results whose accuracy is satisfactory in practice.

## Appendix I

The coefficients in expressions (34) and (35), for $k \leq 4$, are computed from the formulas:

for (34)

$$b_n = (1-\beta)^{k-n} C_k^{k-n} T_j \quad (n = 0, 1, 2, 3),$$
$$b_4 = 1,$$
$$a_0 = b_0 T_0,$$
$$a_n = m_{n-1} + b_n T_0 \quad (n = 1, 2, 3, 4),$$
$$a_5 = 1;$$

for (35)

$$m_0 = (1-\beta)^3 \prod_1^k T_j;$$

$$m_1 = (1-\beta)^2 \{[1-\beta\alpha_1] T_2T_3T_4 + [1-\beta\alpha_2] T_1T_3T_4 + [1-\beta\alpha_3] T_1T_2T_4 + [1-\beta\alpha_4] T_1T_2T_3\},$$

$$m_2 = (1-\beta)\{[1-\beta(\alpha_3+\alpha_4)] T_1T_2 + [1-\beta(\alpha_4+\alpha_1)] T_2T_3 + [1-\beta(\alpha_1+\alpha_2)] T_3T_4 + [1-\beta(\alpha_2+\alpha_3)] T_1T_4 + [1-\beta(\alpha_2-\alpha_4)] T_1T_3 + [1-\beta(\alpha_1+\alpha_3)] T_2T_4\},$$

$$m_3 = \sum_1^4 [(1-\beta)+\beta\alpha_i] T_i = \sum_1^4 [\alpha_i + (1-\beta)(1-\alpha_i)] T_i,$$

$$m_4 = 1.$$

For the case when $k < 4$, the missing time constants $T_i$ and relative impedances $\alpha_i$ must be set equal to zero.

## Appendix II.

Example of the Computations. We shall present the computations for the fall of magnetic flux in the clutch whose magnetic circuit is sketched in Fig. 7, using the following data: $B_{comp}$ = 12,000 gauss; the material is Armco steel; $\mu_{av}$ = 2500, and $\rho$ = 0.15 ohm mm$^2$/m; $\delta$ = 0.35 mm; $b_a = b_d = b_{c1}$ = 0.8 cm; $b_{c2}$ = 1.7 cm.

We now determine the reluctance of the pieces (all lengths are taken along the center lines):

for the rotor and front disk

$$\mu R_a = \mu R_d = \frac{1}{2\pi b_a} \ln \frac{r_2'}{r_1'} = 0.166 \ cm^{-1};$$

for the outer cylinder

$$\mu R_{c1} = \frac{b_{c1}}{\pi d_{c1}' b_{c1}} = 0.1705 \ cm^{-1};$$

for the inner cylinder

$$\mu R_{c2} = 0.1832 \ cm^{-1}.$$

We now find the relative reluctances:

$$\alpha_1 = \frac{2\mu R_a + \mu R_{c1}}{2\mu R_a + \mu R_{c1} + \mu R_{c2}} = 0.733,$$

$$\alpha_2 = \frac{\mu R_{c2}}{2\mu R_a + \mu R_{c1} + \mu R_{c2}} = 0.267.$$

The impedance of the air gap is:

$$R_\delta = 2.6 \cdot 10^{-3}.$$

The equivalent magnetic permeability is:

$$\mu_e = \frac{\mu R_1 + \mu R_2}{R_1 + R_2 + R_\delta} = 238.$$

Now we compute the time constants of the solid members:

$$T_1 = \frac{0.4\pi\mu_e}{10^4\rho}\frac{b_1^2}{\pi^2} = 0.0133 \text{ sec.}, \qquad T_2 = T_1\left(\frac{b_2}{b_1}\right)^2\left(\frac{r_1 + r_2}{r_2}\right)^2 = 0.13 \text{ sec.}$$

The equivalent time constant is

$$T_e = \alpha_1 T_1 + \alpha_2 T_2 \frac{\beta_c}{\beta} = 0.0367 \text{ sec}$$

For $r_2 \geq 1.5 r_1$, we recommend $\beta_c \approx \frac{4}{\pi^2}\frac{r_2 + r_1}{r_1}$.

The parabola parameter is

$$\tau_e = \frac{\pi}{2}(\alpha_1\sqrt{T_1} + \alpha_2\sqrt{T_2}) = 0.283 \text{ sec}^{\frac{1}{2}}.$$

Thus, the process of flux attenuation is described by the curves:

$$\frac{\Phi^*_{off}(t)}{\Phi_0} = 1 - 3.95\sqrt{t}, \qquad \frac{\Phi_{off}(t)}{\Phi_0} = 0.81\, e^{-t/0.0367}.$$

The construction is given on Fig. 6.

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# ON THE DESIGN OF A CONTACT FREQUENCY TRANSFORMER FOR ASYNCHRONOUS MOTOR CONTROL

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The feasibility and efficacy are demonstrated of using a mechanical contact rectifier in the design of a frequency transformer for speed regulation of an ac motor. The properties of a current transformer, working in an inverter regimen, are explained, and methods of solving the commutation problem are set forth.

Of all the methods for regulating the speed of an asynchronous motor, the most highly perfected is that of regulation by variations of the voltage source's frequency.

A number of works [1-4] are devoted to the frequency regulation of asynchronous motors. In them are to be found investigations of the various aspects of the frequency regulation problem, as well as derivations of the most efficient rules for varying the frequency and voltage of the supplying source so as to attain the requisite operating characteristics. The great superiority, both for ease of use and for reasons of economy, of frequency regulation as compared to other forms of regulation (for example, parametric regulation), has been demonstrated.

However, despite the clear superiority of this form of regulation, it has not as yet been widely adopted in industrial practice. The basic reason for this has been the lack of a frequency transformer which is both reliable and economically efficient.

As a source of variable frequency, a transformer must possess a number of properties, the most basic of which are: 1) the ability to control independently the frequency and voltage at its output; 2) the ability to vary frequency smoothly within wide limits; 3) a high efficiency rating; and 4) reliability and convenience in use.

If any of the existing rectifier types is taken as the basis of the frequency transformer, such rectifiers to operate in the inverter regimen, then it becomes necessary to assure satisfactory commutation for all frequencies, for all transitions from one frequency to another, and for various loadings. It becomes necessary, moreover, to assure the virtually independent control of the output voltage and frequency. Moreover, since the desire is to use the frequency transformer for ac motor control in various drive designs, it becomes very important to have the simplest possible means for switching the transformer from the inverter regimen to the rectifier regimen (for example, during motor braking) and vice versa.

The present paper is devoted to the exposition of some preliminary results from an investigation into the design of frequency transformers.

As the basis for our transformer design we chose mechanical, or contact, rectifiers (MR). Today, MRs are built for currents of 20,000 amps and higher for rectified voltages of 400 to 600 volts. Actual usage of contact rectifiers has shown that their contact systems work satisfactorily, without repairs, for eight to ten thousand hours. Mechanical rectifiers possess high efficiency ratings – up to 99.5%. The two-way conductivity of mechanical rectifier contacts gives, in principle, the capability of switching from a rectifying regimen to an inverting regimen and back, reversing both the current and the voltage.

A number of other properties of MRs, making them efficient for use in the design of a frequency transformer, will be considered below.

## Mechanical Frequency Transformers

A mechanical frequency transformer must be comprised of a rectifier portion, transforming ac voltage at industrial frequencies into dc, and an inverter portion, transforming rectified current into ac at the desired frequency. Obviously, the magnitude of the voltage at the inverter output is determined by the voltage at its input, i.e., by the output voltage of the rectifier. Therefore, the solution of the problem of independent regulation of the transformer's output voltage reduces to the solution of the problem of controlling the rectifier's output voltage (whether the rectifier be of semiconductors, mechanical, or something else).

The inverter portion of the design is the most complicated and, moreover, is decisive for the transformer design. Therefore, as the first step in our work we considered the possibilities of designing the inverter portion of a mechanical transformer with the condition that at its input would be applied a dc voltage to be regulated and, in our experimental designs, we used a dc machine as the source of our dc current.

As the basis for the design of the inverter portion we chose a contact apparatus, to be used in the mechanical rectifier, the shaft of which controlled the rotation of the low-power dc motor to be regulated. The frequency of the inverted voltage was thus determined by the speed of rotation of the contact apparatus.

Certain analogies between mechanical contacts in series and saturated chokes and rectifiers allowed the use, as the basis of the electrical portion of the mechanical inverter, of well-known circuits of rectifier inverters.

Below, we shall demonstrate the possibility of designing mechanical inverters under conditions where the system includes forced commutation and interrelated control of a number of parameters.

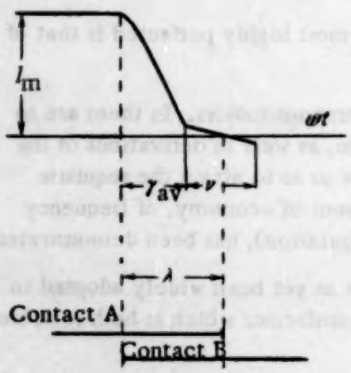


Fig. 1

As was pointed out above, in the development of any inverter, including mechanical ones, it is necessary to satisfy the following requirements: a) the commutation of the transformer contacts in all work regimens must be sparkless; and, b) the form of the transformer output voltage curve must be practically sinusoidal.

### Commutation

Obtaining sparkless commutation of a mechanical inverter, i.e., obtaining sparkless breaking of periodically operating contacts, is possible if the condition is met that, at the moment of contact switching, the current flowing through the contacts is close to, or equal to, zero. In other words, to obtain sparkless commutation, the design must incorporate some supplementary device which periodically decreases, virtually to zero, the current in the contacts to be broken just prior to their switching.

The use of well-known rectifier circuits as the basis of the mechanical inverter permits a decrease, during arcover time, of the current in the contacts to be switched, down to zero and even over into the negative domain.

In theory, if the commutating device were so adjusted, and contact arcover so devised, that the moment of contact switching coincided precisely with the moment of zero current, then the commutation would be sparkless.

In other words, for sparkless commutation in the ideal case, it is necessary that the condition $\gamma = \lambda$ be met, where $\gamma$ is the commutation angle, corresponding to the time during which the current through the contacts decreases to zero from its precommutation value, and $\lambda$ is the angle of contact overlap (Fig. 1).

Under actual conditions, there will always be a certain "play" of the contacts in any contact apparatus (due to slack in the contact apparatus, thermal changes, nonuniformity in contact wear, etc.), and a certain "play" of the commutation angle $\gamma$, arising from the inaccuracies in the functioning of the contact device. Therefore, to obtain sparkless commutation in actual devices, it is necessary to construct a certain spike of low current, the length of this spike being such that contact switching would occur during the duration of this spike even though there might be "play" of the contacts and oscillations of $\gamma$. Such a low current spike can be obtained by the use of a commutating choke with relay characteristics, connected in series with the contacts. With this, the commutating device would have to be so adjusted that, after the current had been decreased, a certain magnetization-switching voltage would be applied to the choke. Then, starting with some small value of current,

the choke would begin to switch its magnetic polarity, as a result of which a "spike" would be formed whose length (duration) would be determined by the choke's parameters and by the magnetization-switching voltage.

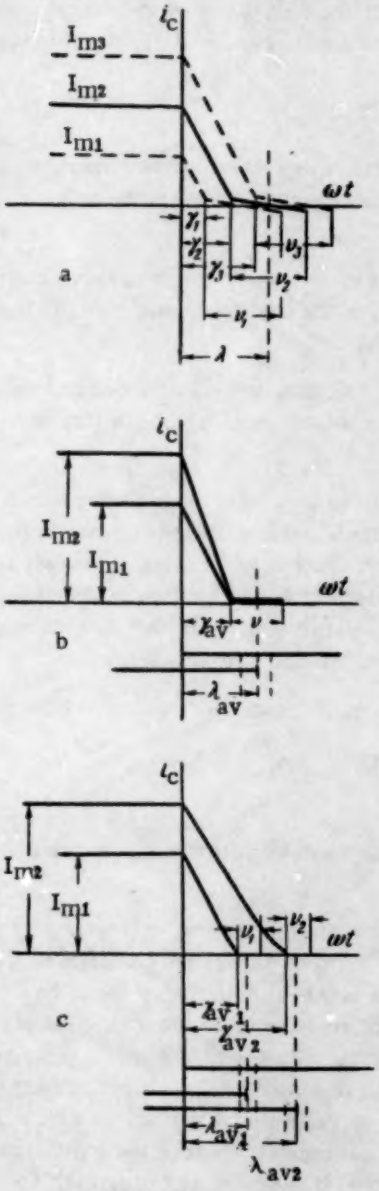


Fig. 2

In this case, reliable commutation would be attained solely by so adjusting the contact overlap that contact switching would always occur during the low current spike.

In other words, in contradistinction to rectifier transformers, where the duration of the simultaneous firing of the two adjacent rectifiers is determined by the magnitude of the angle $\gamma$ and is controlled in a natural way, in a mechanical contact apparatus it is necessary to maintain artificially a definite relationship between the average value of the angle $\gamma_{av}$ and the average magnitude of the overlap $\lambda_{av}$.

This relationship can be written in the form

$$\lambda_{av} = \gamma_{av} + \frac{\nu}{2} , \qquad (1)$$

where $\nu$ is the duration of the low-current spike.

Just as in rectifier transformers, the commutation angle $\gamma$ is determined by the magnitude of the current $I_m$ to be commutated, i.e., by the form of the load, by the design parameters of the device, and by the frequency of the output voltage.

In principle, there are three possible ways of fulfilling the condition for reliable commutation, (1).

1. So to choose the duration $\nu$ of the spike that, for all variations of the commutation angle $\gamma$ and for fixed $\lambda$, the moment of contact breaking does not lie outside of its limits (Fig. 2a). It is on this principle that the majority of contemporary mechanical rectifiers carry out reliable commutation. The disadvantages of this solution are the necessity for choosing a spike of long duration, which leads to a significant increase in the dimensions of the expensive commutating choke, and the limitation of the maximum current $I_m$ by the choke parameters.

2. So to regulate the parameters of the commutating device that with variations of the load or of the output frequency, the average value of the commutation angle $\gamma_{av}$ remains fixed. In this case, the spike must be so chosen that its duration somewhat exceeds the angles corresponding to the "play" of the contacts and the "play" of the commutation angle (Fig. 2b).

Since the duration $\nu$ of the spike depends, not only on the choke's parameters, but also on the voltage which switches its magnetic polarity, variations in the parameters of the commutating device must not lead to variations in this voltage. In this case, the maximum value of $I_m$ is determined by the heating conditions.

3. So to control the magnitude of the contact overlap that, when $\gamma$ varies (Fig. 2c), the following condition will hold:

$$\lambda_{av} = \gamma_{av} + \frac{\nu}{2} . \qquad (2)$$

With this, the problem of the commutating device consists in maintaining a magnetization switching voltage on the choke which will keep the necessary duration $\nu$ of the spike.

Thus, to guarantee sparkless commutation it is necessary, in cases 2 and 3, to introduce continuous control of certain system parameters.

As was stated above, the time $t_{ms}$ for magnetization-switching of the choke and, consequently, the computed width of the spike $\nu$, are determined by the choke's parameters and by the magnitude of the magnetization-switching voltage. The magnetization-switching time can be determined from the following relationship [5]:

$$t_{ms} = \frac{2B_s wQ \cdot 10^{-8}}{e} \quad \text{or} \quad \nu = \frac{2B_s wQ \cdot 10^{-8}}{e}\omega, \tag{3}$$

where $e$ is the magnetization switching voltage on the choke, $B_s$ is the saturation induction of the commutating choke, $w$ is the number of turns of the choke, Q is the cross section of the choke's core, and $\omega$ is the angular frequency.

In the well-known inverter designs, the commutating device (capacitance or master generator) possesses the property that the commutating voltage is related, directly or indirectly, to the device's parameters, to the load, and to the frequency.

Variations of the frequency, load, or commutating device parameters will inevitably lead to variations of the commutating voltage $e$ and, consequently, to variations in the width $\nu$ of the spike which, in turn, could lead to a rupture of the conditions which make for reliable commutation.

In order to render such a rupture impossible, it would be desirable to have a commutating device which would permit independent control of its voltage during the commutation period. Such a device can be obtained by introducing into an ordinary inverter design a special pulse emf generator. Such a generator must satisfy the following basic conditions: 1) the form of its emf must be that of pulses, alternating in sign, whose duration must not be less than $\lambda$; 2) during the intervals between pulses, the output voltage must equal zero; 3) it must be possible to control the pulse amplitude within wide limits on both the positive and negative sides.

Methods of introducing such pulse generators will be shown during the consideration of concrete commutating device designs.

## Reactive Power and Form of the Voltage Curve

It was stated in [6] that the rectifier frequency transformers in ordinary use transmit only the active current components to the recipient device.

Therefore, in cases when the recipient device requires the reactive current components, it is necessary to set up special compensators at the transformer's output, compensators in the form of capacitive batteries or the so-called reactive power machines which, in individual cases, may be more powerful than the motor to be regulated. With capacitive compensation, control power is expended only on rotation of the contact apparatus, and its magnitude does not depend on the reactive power of the recipient device. In the case where a reactive power machine (MFG) has been placed at the transformer's output, variations in magnitude or character of the load invariably lead to variations of the magnitude and form of the current curve in this machine, which could lead to variations in the power consumed in control. In order to guarantee minimum current in the master frequency generator and, consequently, minimum power consumed in its rotation, it is necessary to regulate the emf of the master frequency generator in such fashion that it coincides, in amplitude and phase, with the first harmonic of the voltage on the load. In this case, the only current flowing through the master frequency generator is that determined by the difference between the voltage on the load and the sinusoidal voltage of the same emf, i.e., only the higher harmonics of the current.

## Inverting Design

Figure 3 shows a standard single-phase inverting design with a master frequency, where A and B are contacts of the contact apparatus, DCS is the dc current source, MD is the motor drive of the contact apparatus, MFG is the master frequency generator, $L_f$ is the filtering choke, $L_c$ is the commutating choke and $L_l$ and $R_l$ are, respectively, the inductive and active impedances of the load.

To guarantee sparkless commutation it is necessary so to choose the angle $\alpha$ of contact-making that contact arcover occurs during the time when the commutating current from the master frequency generator is

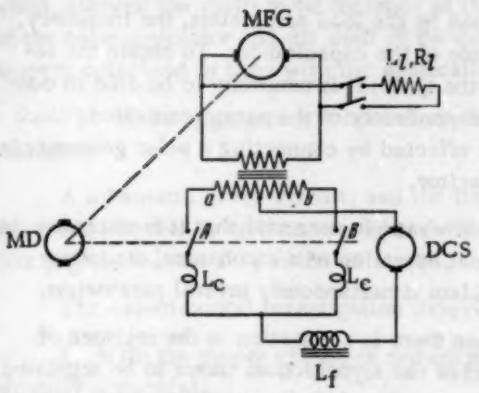


Fig. 3

directed toward the working current in the contacts to be switched. This condition holds during half a period and, consequently, it is theoretically possible to obtain commutation when the angle $\alpha$ varies within the zone $(-\pi/2 + \varphi_{MFG} - \lambda) < \alpha < (\pi/2 + \varphi_{MFG} - \lambda)$ (Fig. 4).

The magnitude of the voltage applied to the choke during commutation is determined by the position of contact arcover with respect to the master sinusoid, i.e., by the contact-angle $\alpha$ and the amplitude of its value. But the condition of obtaining the minimum current in the MFG engenders the necessity of controlling the angle $\alpha$ and the amplitude of the master emf. With this, the magnitude of the commutating voltage and, consequently, the width of the spike, are varied which, in general, leads to a rupture of the conditions for sparkless commutation.

In the design it is possible to include, in series with the master frequency generator, a pulse generator satisfying the conditions cited above, and so tuned that its pulses coincide with contact arcover (Fig. 4). Such a design would allow, by the control of the magnitude and sign of the pulses, the necessary voltage of the commutating device to be obtained. With this, the magnetization switching voltage on the choke will not depend on the phase or amplitude of the master emf. As a result, there will be reliable sparkless commutation of the transformer in any regimen of functioning.

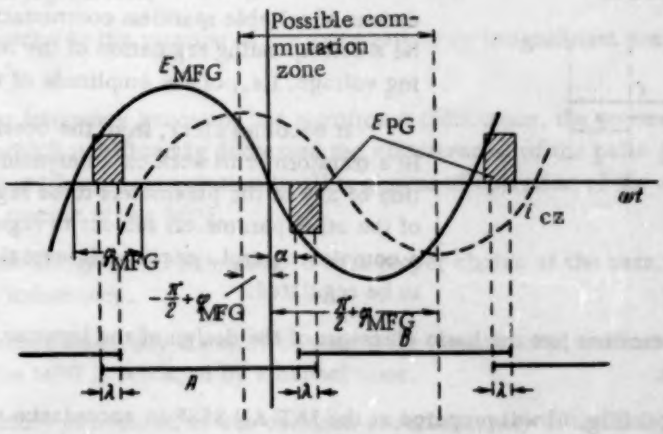


Fig. 4

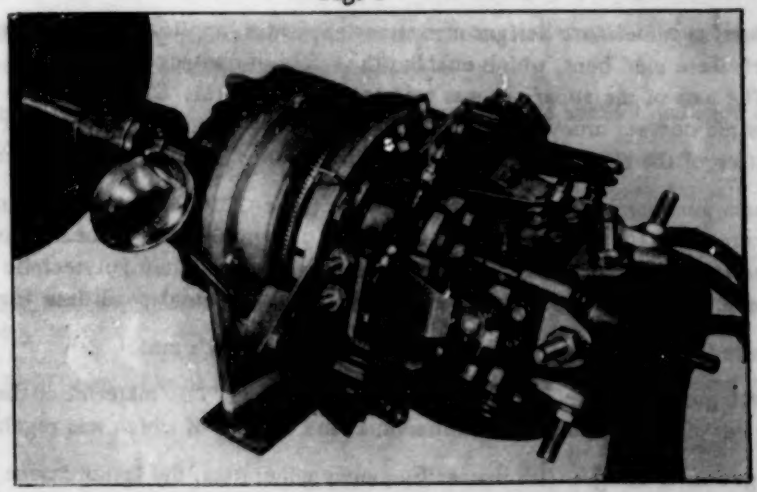

Fig. 5

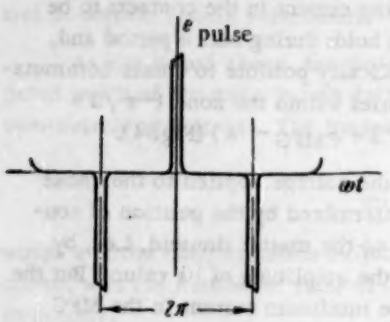


Fig. 6

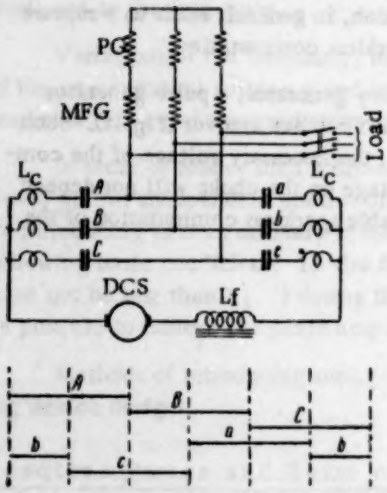


Fig. 7

With capacitive commutation, the voltage on the capacitor at the moment prior to commutation is switched. This voltage is determined by the load parameters, the frequency, and by the magnitude of the capacitance. To obtain the requisite duration of the spike, it is necessary to be able to control this voltage independently of the parameters cited. Such control can also be effected by connecting a pulse generator in series with the capacitor.

It follows from what has been said that it is necessary, in implementing normal operation of a mechanical frequency transformer, to regulate simultaneously several parameters.

Actually, when there is a variation in the regimen of work (speed, torque) of the asynchronous motor to be regulated, it is necessary to change, in a definite way, the output voltage and frequency of the transformer. There occurs with this a variation in the magnitude and character of the load on the transformer.

Corresponding to this, there should be changes in the tuning of the phase and amplitude of the master generator (in the case of an inverter with a master frequency) or in the magnitude of the capacitance (in the case of inversion with capacitive commutation), i.e., the parameters of the commutating device should be changed. Simultaneously with this, so that the conditions of reliable sparkless commutation may hold, there should be a corresponding regulation of the magnitude of the commutating voltage, i.e., of the amplitude of the pulse generator's emf.

It becomes clear, from the consideration of the processes in a transformer to work on an asynchronous motor, that variation of any of the parameters to be regulated leads to variations of the other parameters subject to regulation. The end result is a complex control system with several interdependent quantities to be regulated.

The present paper describes just the basic elements of the design of the inverter portion, and presents some experimental results.

The contact apparatus (Fig. 5) was prepared at the IAT AN SSSR in accordance with a design of the Lenin Polytechnic Institute at Khar'kov. The permissible current across the contacts was 200 amperes.

In contradistinction to ordinary designs of contact apparatus, applicable to mechanical rectifiers, worm gears and handwheels were used here, which enabled the angle of contact making to be varied during operation of the apparatus. The axis of the apparatus was maintained horizontal. Changes in the contact time, i.e., the establishment of desired contact arcover, was done at the forward portion of the contact apparatus. This device permitted the variation of the relative positions of the immovable and movable contacts.

The three-phase generator of sign-alternating pulses (type GZI-1) has a maximum voltage of 110 volts, a maximum pulse current of 100 amperes, and a frequency of from 30 to 50 pulses/second. The pulse duration is from 15 to 20 electric degrees. The generator was prepared by the Lenin Polytechnic Institute at Khar'kov with a type PN-85 machine as the base, in accordance with the technical conditions imposed by the IAT AN SSSR.

The oscillogram (Fig. 6) shows the curve of the pulse generator's emf.

The commutating chokes were prepared at the IAT AN SSSR. The material of their cores was KhVP. As the dc source there was used a type PN-100 dc machine, the voltage of which was regulated.

In the investigation of designs with master frequency generators, the latter device was implemented by a synchronous generator with sharply defined fields of 4.5 kilowatt power.

The shaft of the MFG and the shaft of the pulse generator were mechanically linked by means of a clutch, which allowed the shafts to be matched at the required angle. There was a rigid connection between the shaft of the pulse generator and the shaft of the contact apparatus. This connection was so established that contact arcover coincided in time with the application of the pulses.

The whole aggregate was hooked up to the rotation of the machine to be regulated, a type PN-68 dc machine.

A schematic of the circuit, and the timing diagram for the contact operations, are shown in Fig. 7. The following notation is used on the figure: PG is the pulse generator, $L_c$ are the commutating chokes, $L_f$ is the filtering choke, and DCS is the dc source.

The experimental investigation showed the following:

1. With the proper choice of system parameters, the voltage and current curves at the inverter output are virtually sinusoidal.

2. With variations of the load of the asynchronous motor to be regulated, it is necessary so to regulate the contact-angle $\alpha$, the amplitude of the master frequency generator's emf $E_{MFG}$, and the emf of the dc source $E_{DCS}$ that the following conditions hold:

$$\alpha = \varphi_l \quad \text{и} \quad \frac{E_{MFG}}{E_{DCS}} = \frac{2\sqrt{3}z_l}{3x_l + \pi R_l},$$

where $z_l$ is the total load impedance, $x_l$ is the inductive load impedance, $R_l$ is the active load impedance, and $\varphi_l$ is the angle of phase shift between the load current and voltage. With this, one attains minimum current in the master frequency generator.

3. The power consumed in the rotation of the aggregate is an insignificant portion of the transformer's output power.

4. Since the master frequency generator had significant inductance, the process of commutating current increase was protracted, which significantly decreased the effectiveness of the pulse generator and led to the impossibility of obtaining sparkless commutation in all regimens of operation of the asynchronous motor being regulated with minimum current in the MFG.

The usefulness of the design could be enhanced by a proper choice of the parameters of the MFG, specifically, by decreasing its inductance.

The design of a contact frequency transformer with pulse-capacitance commutation differed from the one described above in that the MFG is replaced by a capacitance.

Completely satisfactory commutation was obtained experimentally in controlling an asynchronous short-circuited motor of 7 kilowatt power, both free-running and underload, in the range of 6 to 60 cycles.

With this, when the speed of the regulated asynchronous motor was varied, the pulse amplitude, the magnitude of contact arcover and the capacitance were regulated.

With this design it turned out to be possible to carry out starting and speed regulation of an asynchronous motor of 28 kilowatt power when it was running without load.

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# MULTIFREQUENCY PULSE DEVICES FOR TELECONTROL

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(Kiev)

The basic principles are presented for electromechanical vibrators with pulsed excitation which are used to form the frequency signals for telecontrol and telemetering multifrequency devices (TC-TM). Variations of the TC-TM design are given for the case when the objects are distributed along the common communication channel.

Multifrequency devices for telecontrol, whose principles of design were presented in [1], require for their operation sources of frequency signals which possess significant stability.

The requirements on the stability of the frequency signal generator depend on the bandwidth for operating the relay, account being taken of bandwidth variations. The admissible instability of the frequency generator can be determined from the expression

$$\delta f_r = \pm \frac{\Delta f_{op} - 2\delta f_{re}}{2K_s},$$

where $\Delta f_{op}$ is the bandwidth for relay operation, $\delta f_{re}$ is the shift of relay operation bandwidth when external conditions (temperature, voltage, etc.) vary, and $K_s$ is the safety factor, lying between 2 and 3.

For use in multifrequency TC-TM devices with narrow-band electromechanical frequency relays having operation bandwidths of two to three cycles, frequency generators are subject to such high stability requirements that only electromechanical vibrators should be employed in their construction. Frequency signals are obtained most simply from them by virtue of their pulsed excitation. For this, a condenser is discharged across the input winding at the proper moment, giving rise to a frequency pulse at the vibrator output with a gradually damped amplitude. For telecontrol purposes, this pulse is given preparatory amplification.

When connecting cables are to be used, where the noise level is low, the frequency signals obtained from the vibrator output may be passed directly into the communication channel without preliminary amplification, for subsequent use in telemetering.

For this case, we determine the distance of signal transmission along the connecting cables. As is known [2], the noise level in cables with no wireless links equals −7 to −8 nepers in a 5 kilocycle bandwidth.

Since the band used by the given TC-TM devices does not exceed 200 cycles, it is possible to choose a lower noise level for the computations. However, we will start with the average value of −7.5 nepers.

To guarantee reliable operations of the TC-TM frequency device, it suffices to have an excess of 2 nepers of minimum signal level over average noise level. Consequently, the minimum signal level at the amplifier input must be −5.5 nepers.

By taking into account the exponential character of frequency pulse damping at the vibrator output, we may write the expression for the power generated by the vibrator during time $t$ after excitation in the form [3]

$$P_F = P_I e^{-d_d \omega_0 t},$$

where $P_I$ is the initial power at the vibrator output, $d_d$ is the damping coefficient in the vibrator system, and $\omega_0$ is the angular frequency of resonance.

To determine the minimum value of power at the vibrator output, we should set $t = 1.5\,t_{op}$, where $t_{op}$ is the operating time of the receiving frequency relay.

If we let $f_0 = 160$ cycles, $d_d = 0.01$, $P_I = 1.5 \cdot 10^{-3}$ watts, and $t = 0.3$ seconds, then $P_F = 7.35 \cdot 10^{-5}$ watts.

If we take into account the mutual shunting of the vibrators for two-frequency signals, then $P'_F = P_F/2 = 3.67 \cdot 10^{-5}$ watts, or $-5.1$ nepers

For noncoil-loaded telephone lines of 1.2 mm diameter the kilometric damping, for a frequency of 300 cycles, is $\beta_{300} = 0.031$ nepers/kilometer. By bearing in mind that the damping is proportional to the square root of the frequency, we obtain, for a frequency of 160 cycles, $\beta_{160} = 0.023$ nepers/kilometer. Consequently, in this case the transmission distance will be

$$d = \frac{5.5 - 5.1}{0.023} = 17.4 \text{ kilometers}$$

For higher frequencies, the transmission distance will be correspondingly smaller. In order that the combining frequencies, created when multifrequency signals enter the amplifier input, do not give rise to additional noise, the amplifier should be tuned to the working frequency band which is defined by the expression $f_u = 2(f_l - \Delta f_{re})$, where $f_u$ and $f_l$ are, respectively, the upper and lower frequencies of the working band, and $\Delta f_{re} = (2 \text{ to } 5)\Delta f_{op}$ is the difference frequency of the working frequencies.

If the noise level in the channel is high and more than two frequencies are simultaneously transmitted, then an amplifier should also be used on the transmitting end. These amplifiers are prepared from transistors according to standard circuits.

When vibrators are used as pulsed frequency generators, it is important to determine the length of the exciting pulse for which the initial amplitude of the free oscillations will be maximal. To simplify the analysis, we shall assume that the pulses are rectangular, of width $\tau$.

When the leading edge of the pulse is operative, the equation of the free oscillations will have the form

$$X(t) = -A_I e^{-\frac{1}{2} d_d \omega_0 t} \sin \omega_0 t,$$

where $A_I$ is the initial amplitude of the oscillation.

At time $t = \tau$ the free oscillations corresponding to the trailing edge of the pulse arise. The initial amplitude of these oscillations will have a sign opposite to that of the initial amplitude of the oscillations arising when the leading edge of the pulse impinged. Consequently,

$$X(t-\tau) = A_I e^{-\frac{1}{2} d_d \omega_0 (t-\tau)} \sin \omega_0 (t-\tau).$$

The resulting oscillations, for $t > \tau$, will be

$$X(t) + X(t-\tau) = A_I e^{-\frac{1}{2} d_d \omega_0 t} \left[e^{\frac{1}{2} d_d \omega_0 \tau} \sin \omega_0 (t-\tau) - \sin \omega_0 t\right].$$

From this expression we can determine the greatest amplitude of the oscillations for $t > \tau$. For this, and bearing in mind that $d_d \ll 1$, and $\tau < T_0 = 2\pi/\omega_0$, we set $e^{\frac{1}{2} d_d \omega_0 \tau} \approx 1$. Then, the expression for the resulting oscillations can be put in the form:

$$X(t) + X(t-\tau) = -2A_I \sin \omega_0 \frac{\tau}{2} \left[e^{-\frac{1}{2} d_d \omega_0 t} \cos \omega_0 \left(t - \frac{\tau}{2}\right)\right],$$

and the desired maximum amplitude of the oscillations will equal

$$A_m = 2A_I \sin \omega_0 \frac{\tau}{2}.$$

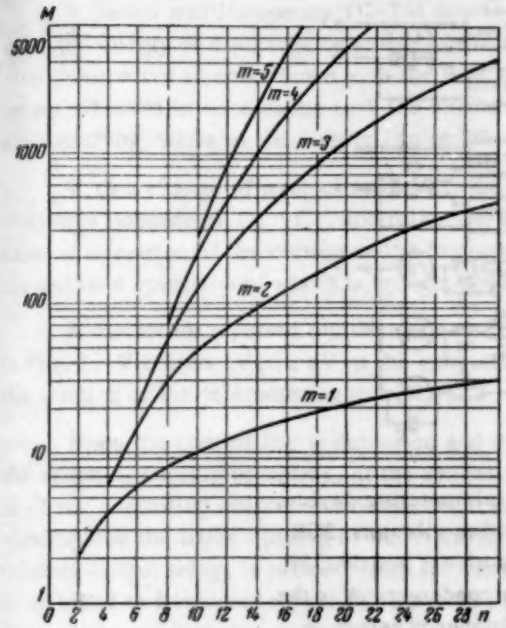


Fig. 1. Curves for determining the number of frequencies m in a multifrequency signal as a function of the required number of commands N and the number n of working frequencies.

We shall now determine the pulse length $\tau_m$ for which the resulting oscillations will be maximal. For this, we differentiate the equation for $A_m$ with respect to $\tau$, and equate the result to zero:

$$\frac{dA_m}{d\tau} = A_1\omega_0 \cos\omega_0\frac{\tau_m}{2} = 0.$$

Remembering that $\tau < T_0$, we get $\tau_m = T_0/2$.

When the vibrator is excited by pulses from a condenser discharge, the process of setting up the oscillations occurs in a much more complicated fashion, since the rise and fall of the pulse do not occur instantaneously. There is importance also in the phenomenon of energy exchange and, hence, additional dissipation, between the condenser and the vibrator. It has been established experimentally that the optimal capacity of the discharge condenser somewhat depends on the vibrator load, in such fashion that a decrease of load impedance increases the optimal value of the discharge condenser's capacitance. This phenomenon can be explained by the fact that a decrease of load impedance lowers the time constant of the condenser discharge circuit, since the equivalent discharge impedance is lowered.

It should be mentioned that the dependence of the initial amplitude of vibrator oscillations on the capacitance of the discharge condenser, when close to the optimal value, is not significant, which renders possible the use of one discharge condenser for several vibrators close to the tuned frequency.

In constructing a multifrequency TC-TM system, it is very important to determine the number of frequencies to be transmitted simultaneously. For this, it is convenient to use the curves shown in Fig. 1. It is necessary to bear in mind with this that an increase in the number of frequencies entering into the multifrequency signals increases the noise stability of the TC-TM device on the one hand and, on the other hand, increases the unit number of contacts and increases the power dissipation at the receiving amplifier's output.

The increase of noise stability of a multifrequency code over a single-frequency one, with independent source deviations, can be determined on the basis of the following considerations.

We shall assume that the probability of incorrect operation of one of n frequency receivers (frequency relays) equals P. Then, for a single-frequency code, the total probability of incorrect operation for a set of N two-position objects under control, equals

$$P_1 = 2NP,$$

and for a multifrequency code,

$$P_m = 2N(K_P P)^m,$$

where $K_P$ is a coefficient which expresses the fact that there is an increase in the probability of an incorrect operation of the individual receiver relays due to the decrease in power of the signal components of the multifrequency signal with the limitation on their mean level, and m is the number of frequencies simultaneously transmitted.

From a comparison of these two expressions one may draw the conclusion that the use of a multifrequency code, for the same number of controlled objects, varies the noise stability by a factor of $(K_P^m P^{m-1})^{-1}$.

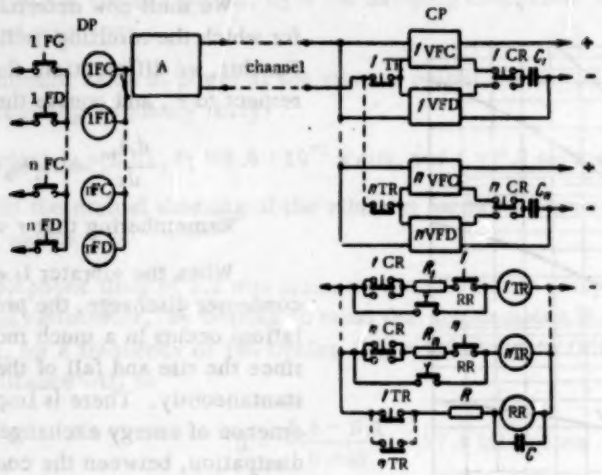


Fig. 2. Circuit for a pulsed multifrequency telemetering device. 1VFC..nVFC...nVBD are telemetering position vibrators, 1CR ... nCR are contacts of the relay controlling object position, RR is the release relay, $C_1 \ldots C_n$ are discharge condensers, A is the amplifier, 1FC . . . nFC, 1FD . . .nFD, are frequency relays, R . . .$R_n$ are additional impedances.

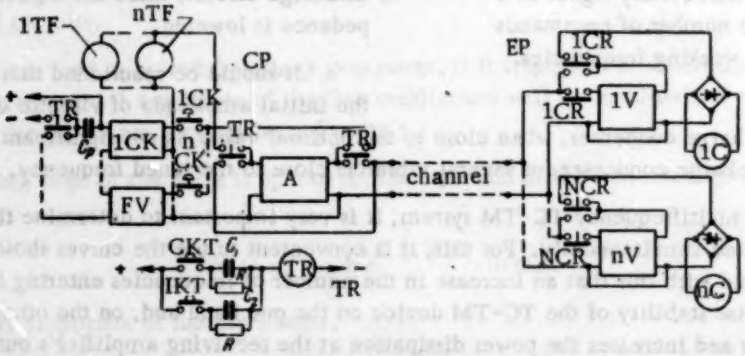


Fig. 3. Circuit for a pulsed multifrequency TC-TM device with telemetering based on the energy of the control pulses. CP is the control point, 1TF . . .nTF are frequency relays receiving telemetering pulses, 1FV . . .nFV are vibrators forming the control pulses, A is the amplifier, TR is the trigger, relay, 1CK . . . nCK are the control keys, $C_1$, $C_2$ and $C_p$ are condensers, CKT is the control knob trigger, IKT is the interrogation knob trigger, 1V . . . nV are vibrators of the TC-TM, 1C . . . nC are the output control relays, and 1CR . . . nCR are contacts of the relays for controlling the state of the objects.

Insofar as $K_p \approx m$ and $P \ll 1$, the increase of noise stability obtained by using a multifrequency code quite overshadows the decrease in noise stability of the individual receivers due to the decrease in power of the components of the multifrequency signals. On balance, the noise stability of the system is increased, as has been verified experimentally.

The average, over one period, of the power of the multifrequency signal at the input to the set of n frequency relays, connected in parallel, must equal

$$P_{ap} = mnP_{fr},$$

where $P_{fr}$ is the power required by one frequency relay.

The input amplifier must be designed for this power .

A pulsed multifrequency TC-TM device can be built in two basic variants: with vibrator excitation rom the energy of discharge condensers or from the energy of control pulses. A circuit for a telemetering device constructed in accordance with the first variant is shown in Fig. 2. At the controlled point CP there are t up vibrators in accordance with the number of telemetering positions and individual control relays and signal switching, while at the dispatch point DP are the amplifier A and the frequency relays.

With a change of state of an object, the contacts of control relay CR are switched into the circuit of the ischarge condensers, $C_1 - C_n$, and relay TR, and a telemetering frequency pulse is sent through the channel causing operation of the corresponding frequency relay at the DP. Relay TR is released after relay RR operates, the delayed operation of which is brought about by the presence of resistor R and condenser C.

A circuit for a pulsed multifrequency TC-TM device built in accordance with the second variant is shown in Fig. 3. Vibrators 1V. . . nV at the executive point are used both for reception of the control pulses and for the sending of the telemetering pulses.

When the control key is depressed and the control knob trigger is pressed, a frequency pulse is sent along the channel, causing operation, at the executive point EP, of the corresponding frequency relay and switching in of the controlled object. With this, the control relay CR switches its contacts into the circuit of the vibrator winding and the input winding of the vibrator is switched out of the channel while the output winding, with the winding output relay, is switched into the channel. When relay TR loses its excitation, the output of amplifier A is switched to frequency relays 1TF . . . nTF, and its input is switched into the channel. With this, the oscillations of the vibrator do not immediately cease and, on the EP side of the channel, there will be sent a frequency signal which, after being amplified, will act on the corresponding frequency relay.

When the interrogation knob trigger IKT is pressed, the duration of the frequency pulse in the channel is not sufficient to cause operation of the output relay (since $C_1 > C_2$) and, consequently, it becomes possible to interrogate the state of the objects.

A description of the device and of the characteristics of the frequency relays which were used in constructing the TC-TM devices considered here are given in [4]. There is therein also given a description of the complete circuit, and experimental results, for a pulsed multifrequency TC-TM device for centered objects, the experimental model of which was built by the Department of Automation and Remote Control of the Kiev Polytechnic Institute. The same group developed a pulsed multifrequency TC-TM device for objects distributed along a common communication channel. The principles underlying the construction of such devices are given below.

Depending on the concrete requirements, one may create many variations of such types of devices.

For a comparatively small number of executive points and objects at them, one might recommend a direct multifrequency combined set of objects at the individual points.

If the number of executive points is large, or the number of objects at them is large, then the set of executive points should first be isolated, after which the object should be isolated. In order, in such a scheme, to increase the rapidity of action and to simplify the circuitry, the communication channel should carry a multifrequency signal consisting of a combination of the frequency to isolate the executive point and the frequency to isolate the object. Only the frequency relays for isolating the executive point (group selection relays) should be directly connected to the channel and, upon their operation, the proper block of frequency relays for object selection will be connected to the channel. In certain cases, in order to reduce the power dissipated by the group selection relays, it makes sense to connect only one of the group relays to the channel and let its operation switch the remaining group relays into the channel.

Telemetering from dispersed objects is carried out in a somewhat more complicated manner than telecontrol. Use may be found for the following three variants: simultaneous telemetering, serial telemetering, and telemetering by interrogation.

In the variant with simultaneous telemetering, signals may, as they arise, be accepted simultaneously from several points. Since, in this case, signals may accumulate one on another, they must have different

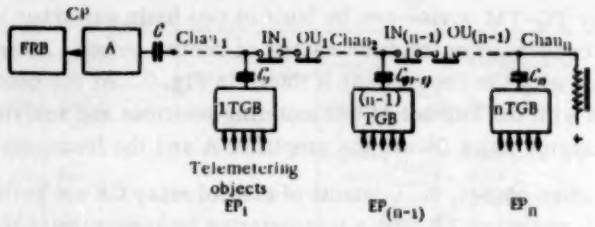


Fig. 4. Block schematic of a multifrequency device for serial telemetering.

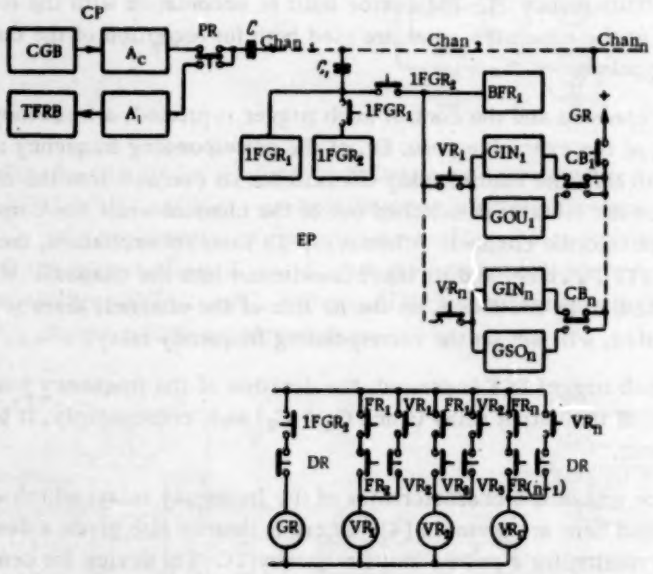


Fig. 5. Circuit for a multifrequency device with telemetering by interrogation. $A_C$ and $A_t$ are amplifiers, CGB is the control generator block, TFRB is the telemetering frequency relay block, $CB_1$ and $CB_n$ are object contact blocks, and DR are deblocking relay contacts.

frequencies. Moreover, the input amplifier must have sufficient power at its output to operate simultaneously the frequency relays equal in number to the maximum possible number of frequency signals at the input. The adjustment of the receiving frequency relays must take into account that, with the minimum number of frequency signals at the output and their power increased at the amplifier output, there does not occur overlap of the bands of relay operation.

It is clear that the variant just considered can be recommended when there is a comparatively small number of executive points and objects at them. However, their number can be increased if the most important executive points are given the more "active" codes. This idea can most simply be implemented by so increasing the power of the signals from a part of the executive points that the signals from these points, when passing through the amplifier, will suppress signals from the remaining points. After acceptance of the signals from the "active" executive points, the signals from the remaining points may be accepted, the duration of the signals having to be correspondingly greater. Several such levels may be used.

In the variant with serial telemetering, signals may be accepted at each moment only from one of the executive points, where the points closest to the control point are given preference in the telemetering. For this, when there is telemetry from an executive point situated close to the control point, all of the tail of the communication circuit is cut off. After the signal is reproduced at the control point, the latter sends an acknowledgement (quitting) signal which causes the channel to be reestablished, allowing telemetering signals from the executive point next farther removed from the control point to be received. Until reestablishment of the channel

to the more remote points, the telemetering signals must be stored. The high speed of the device makes it possible to obtain telemetry signals from the most remote executive points even when there is a significant number of them.

A block schematic for such a device is shown in Fig. 4, where each TGB is a telemetry generator block and FRB is the receiving frequency relay block.

With variation of the state of an object, all of the succeeding portion of the channel is cut off by contacts of the position relays IN or OU until a quitting signal is received from the CP. Control of the channel is carried out by means of a dc source on the side of the later EPs.

Interrogation of object state in the variants so far considered may be carried out most simply by using a telecontrol device for switching in and out the corresponding objects.

In the variant with telemetering by interrogation, data is initially taken from the executive points by telecontrol, and is thereafter obtained by interrogation. Transmission may be carried out, as desired, either by dispatcher control or cyclically after a definite time interval. The principles underlying the construction of such a cevice are shown in Fig. 5. Relay PR is normally open, and amplifier $A_t$ is connected to the channel. At interrogation, relay PR operates and a two-frequency signal is transmitted from the CP, selecting the EP, at which the corresponding frequency group relays $FGR_1$, then $FGR_2$, operate. Relay $FGR_2$ is held and switches block frequency relay BFR into the channel, but contacts of relay GR connect the source to generators GSV and GSO. After this the CP sends successive combinations of selection frequencies to the interrogated objects, at which are operated, and held, relays VR. These relays switch into the channel the corresponding pairs of telemetering generators GIN and GOU. With this, the given EPs send frequency pulses along the channel, thus defining the state of the interrogated objects. These pulses are received at the CP and act on the previously prepared signal circuits. What this preparation amounts to is that, in the time intervals between transmissions of selecting combinations, relay PR is released and the apparatus at the CP is switched over to signal reception.

The deblocking (releasing) of group relay $FGR_2$, after interrogation of the following EP, can be implemented by the transmission of a special combination frequency or, if a remote source is used, by a brief breaking of the supply channel.

In the circuit considered, interrogation of object states may be carried out episodically, by depressing of the corresponding keys by a dispatcher, or cyclically by means of a specially programmed switch. It is also possible to construct the device in such a way that, with a spontaneous change of state in one of the objects, a warning signal is sent to the CP, this signal either ringing a bell or automatically starting up the programmed switch.

With a comparatively small number of objects, interrogation of states may be carried out by excitation of the individual vibrators by pulse control (Fig. 3).

TC-TM devices, constructed on the basis of the principles considered here, may find use in the remote control of many processes in the various branches of the Soviet Union's economy.

A further problem is the development of a contactless multifrequency TC-TM device.

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# A PRECISION MULTIPLIER

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(Moscow)

A description is given of a multiplier with a static error not exceeding 0.01 to 0.02% of the full scale of output voltage, and with the ordinary high speed of electronic components. The low value of error is made possible by constructing the multiplier of a "rough" and an "exact" part.

## INTRODUCTION

The necessity for multiplying two voltages arises in the design of nonlinear models of dynamic systems [1], in certain electrical computing instruments (harmonic analyzers, correlators, etc.) [2], and in a number of cases in the automation of productive processes. A device which carries out this operation is called a multiplier (abbreviated MU).

In most widespread use today are pulse-time multipliers [3], diode multipliers [4], and multipliers with carborundum resistors [5,6]. The errors of these multiplier types lie within the limits of 0.2 to 0.5% of the output scale, and seem to be incapable of essential reduction.

In many cases, the error just mentioned is inadmissible, thus giving rise to the necessity of more accurate MU's. The accuracy of an MU can be increased by using compensatory methods in its circuit. This was the path followed by Goldberg [7] and by Morril and Baum [8]. In designing pulse-time MU's, these authors used precision clutches, based on the compensation principle, which they had developed.

However, these circuits were comparatively complex, and their adjustment was difficult. There is another way to increase the accuracy of an MU, based on the use of the discrete principle. The first to take this path was the American investigator Goldberg [9].

We shall consider below the circuit for a precision multiplier which was presented by A. A. Fel'dbaum and the present author in 1952 [10], this multiplier differing from Goldberg's both in its underlying principle and in its quality. It is simpler than the American circuit, containing fewer elements and being easier to adjust.

The circuit to be discussed differs from the American circuit in that it does not require the use of very fast electromagnetic relays whose contacts function in a regimen of high-frequency vibration and, therefore, rapidly wear out.

In a number of instances, the dynamic properties of the MU design,to be described below,turn out to be significantly better than those of Goldberg's design, particularly in its transfer function for large deviations.

## A Multiplier with "Rough" and "Exact" Parts

The precision MU to be discussed, the block schematic of which is shown in Fig. 1, may be divided into two parts. We shall call them the "rough" and the "exact" portions.

One of the input voltages $x_1$ is applied to the input of the separator block. Here voltage $x_1$ is separated into two portions, the basic portion $\overline{x}_1$ and the supplementary portion $\Delta x_1$. There is a number of fixed values of voltage present in the separator block. That one of them which is closest in value to voltage $x_1$ is taken as the basic portion $\overline{x}_1$ which is applied to the input of the rough system.

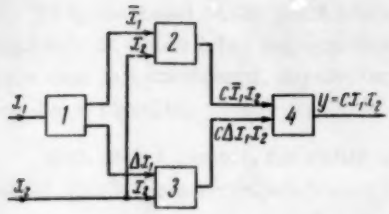


Fig. 1. 1) Separator block; 2) "rough" portion; 3) "exact" portion; 4) summing block.

The second, supplementary, portion

$$\Delta x_1 = x_1 - \bar{x}_1 \qquad (1)$$

is applied to the input of the exact portion. In the rough portion, voltage $\bar{x}_1$ is multiplied, with high accuracy, by the second factor $x_2$.

The name "rough" is given to this portion only because one of its factors, $\bar{x}_1$, differs by $\Delta x_1$ from the factor $x_1$.

Voltage $\Delta x_1$ is applied to the exact portion. Here it is first amplified with gain $\underline{k}$, then multiplied by factor $x_2$ using an MU of some standard type, for example, one of those mentioned above, and then, finally, attenuated by the factor $\underline{k}$.

At the multiplier output there is a summing block for adding the output voltages of the rough and exact portions. In this case, the output voltage will be

$$y = cx_1x_2 = c\bar{x}_1x_2 + c\Delta x_1x_2. \qquad (2)$$

Here, $\underline{c}$ is the coefficient of proportionality of the multiplier. For $(x_1)_{max} = (x_2)_{max} = 100$ volts and $(y)_{max} = 100$ volts, $\underline{c}$ will equal 0.01.

Let us now consider the reasons which allow the error in such a combination-type MU to be lowered.

The output voltage of the MU is

$$y = cx_1x_2 + \delta. \qquad (3)$$

Here $\delta$ denotes the error in multiplication. This error can be represented as the sum of three components:

$$\delta = \delta_s + \delta_r + \delta_e, \qquad (4)$$

where $\delta_s$ is the error of the summing block, $\delta_r$ is the error of the rough system, and $\delta_e$ is the error of the exact system.

If, for the summing block, one uses an operational amplifier with automatic zeroing [11] and with wire resistors in the feedback path, the error $\delta_s$ can be ignored.

The error $\delta_r$ is also negligible, since multiplication in the rough portion is usually implemented, as will be shown below, by switching of the impedance at the input of the summation block by means of relay or selector contacts. Thus, the error of the combination-type multiplier is close to the error of its exact portion:

$$\delta \approx \delta_e. \qquad (5)$$

The maximum value of the reduced voltage $\Delta x_1$ from the output of the separator block is several times smaller than the maximum value of the multiplier's input voltage $x_1$.

Let

$$(x_1)_{max}/(\Delta x_1)_{max} = n. \qquad (6)$$

At the input of the exact system, $\Delta x_1$ is amplified by a factor of $\underline{k}$. The gain $\underline{k}$ is chosen to be close to $\underline{n}$ (ordinarily, $k \leq n$). The amplified voltage $k\Delta x_1$ is applied to the input of an ordinary electronic MU with an error of $\delta_0$. The voltage at the output of this MU is

$$y^* = kc\,\Delta x_1 x_2 + \delta_0. \qquad (7)$$

A voltage divider at the output of the exact system effects a k-fold decrease of $y^*$.

As a result of these operations, the input of the MU's summing block receives from the exact portion's output the voltage

$$\bar{y} = c\Delta x_1 x_2 + \frac{\delta_0}{k}. \qquad (8)$$

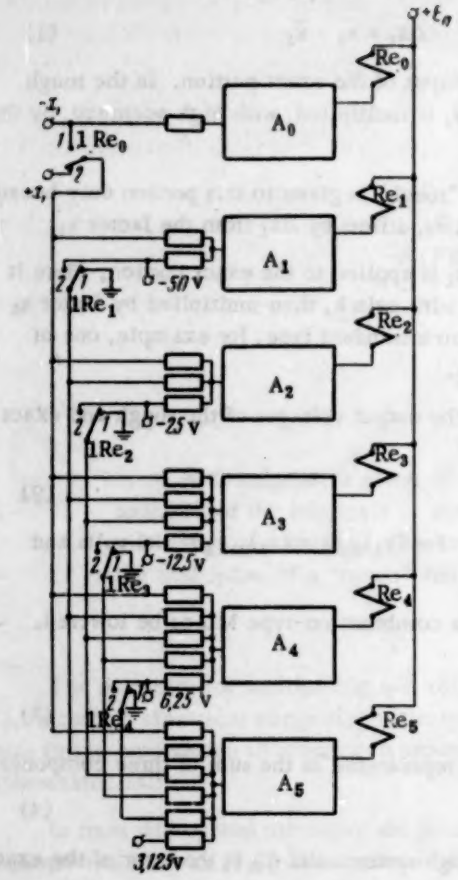


Fig. 2

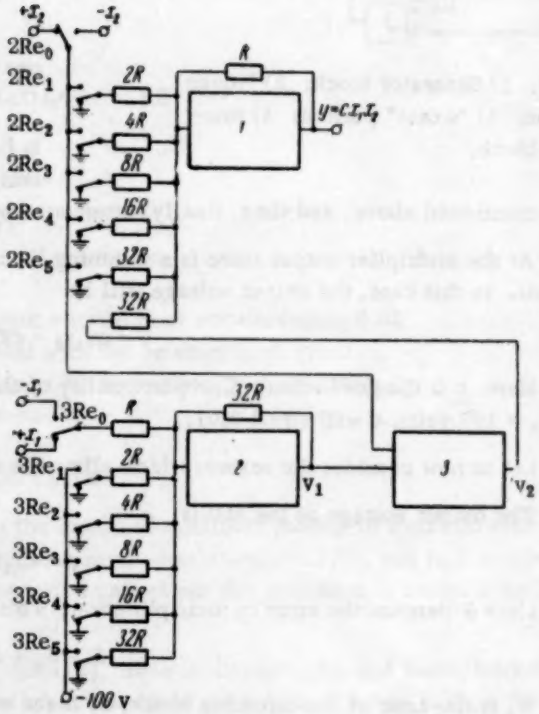


Fig. 3

Since

$$\delta_e = \frac{\delta_0}{k}, \quad \text{a} \quad \delta \approx \delta_e,$$

we have that

$$\delta \approx \frac{\delta_0}{k}. \tag{9}$$

Consequently, the error of the combination-type MU is lower by a factor of k than the error of an MU of the ordinary type.

## Multiplier with a Relay-Type "Rough" Portion

Figure 2 shows the rough portion of a combination-type multiplier consisting of electromagnetic relays $Re_0$ to $Re_5$, and high-gain dc amplifiers $A_0$ to $A_5$. The relay coils are connected in the anode circuits of the amplifiers' output stages.

In the circuit given, switching of the contacts of relay $Re_0$ is determined by the sign of input voltage $x_1$; the positions of the contacts of relays $Re_1$-$Re_5$, connected in series with amplifiers $A_1$-$A_5$ with several input voltages, are determined by the signs of the sums of these voltages. Contact $1Re_0$ is so wired in that $x_1$, with positive sign, is always applied to one of the inputs of each of amplifiers $A_1$-$A_5$.

With the resistances at the inputs of amplifiers $A_1$-$A_5$ all equal, relay $Re_1$ operates for $|x_1| > 50$ volts. Relay $Re_2$ operates with $|x_1| > 25$ volts but, in the case when $|x_1| > 50$ volts, relay $Re_1$ operates and its contact $1Re_1$ switches −50 volts to one of the inputs of each of amplifiers $A_2$-$A_5$, thanks to which the contacts of relay $Re_2$ are opened.

If $x_1$ increases to the point when $x_1 - 50 > 25$ volts, the total voltage on amplifier $A_2$ again reaches the magnitude at which relay $Re_2$ operates. The states of the relays in the circuits of amplifiers $A_3$, $A_4$ and $A_5$, as in the case just considered, depend on the value of $x_1$ and on the states of the contacts of the relays in the amplifiers' input circuits.

With such a circuit, the entire range of variation of $x_1$ (± 100 volts) is divided into different intervals to each of which there corresponds a definite combination of the relay contacts.

For example, let $x_1$ = 78 volts. In this case, contact $1Re_0$ will be in state 2, since $x_1 > 0$. Contact $1Re_1$ will be in state 2 since $x_1 > 50$ volts, and contact $1Re_2$ will also be in state 2 since $x_1 - 50 > 25$ volts. Since $x_1 - 50 - 25 < 12.5$ volts, contact $1Re_3$ drops into state 1.

Obviously, from what has been said, contact $1Re_4$ drops into state 1. Relay $Re_5$ will be in the same state as relay $Re_4$. No contacts of relay $Re_5$ appear on the portion of the MU circuit shown. Instead of different magnitudes of voltage (−50 volts, −25 volts, −12.5 volts, −6.25 volts, −3.125 volts) at the amplifier inputs, one might have applied equal voltages, but across unequal resistances.

The second portion of the MU we are considering is shown in Fig. 3. Here, 1 and 2 are amplifiers and 3 is a multiplier of some standard type. By means of contacts of relays $Re_0$-$Re_5$ (Fig. 2), resistors are switched into the inputs of these blocks. In dependence on the sign of $x_1$, contact $2Re_0$ of relay $Re_0$ switches either voltage $+x_2$ or $-x_2$ to the input of amplifier 1 in such a way that the product $y$ has the proper sign.

Voltage $x_2$ can be applied to the amplifier inputs across resistances of 2R, 4R, . . . , 32R.

At the input of amplifier 2 a voltage of −100 volts is impressed across resistances of 2R, 4R, . . . , 32R and contacts of the relays mentioned, as well as voltage $x_1$ across resistance R.

In the example given, with $x_1$ = 78 volts, voltage $x_2$ is applied to amplifier 1 via contacts $2Re_1$ and $2Re_2$ and resistances 2R and 4R. Simultaneously, a voltage of −100 volts is applied to amplifier 2 via contacts $3Re_1$ and $3Re_2$.

Voltage $x_1$ is applied to the input of this same amplifier. A resistance of 32R is connected between the input and the output of the amplifier.

Consequently, the voltage at the amplifier's output is

$$u_1 = 32x_1 - 100\left(\frac{32}{2} + \frac{32}{4}\right) = 96 \text{ v.}$$

For $x_2$ = 100 volts, the voltage at the output of block 3 is

$$u_2 = 0.01 \times 96 \times 100 = 96 \text{ volts}$$

The voltage at the MU's output is

$$y = \frac{1}{2}x_2 + \frac{1}{4}x_2 + \frac{u_2}{32} = 78 \text{ v.}$$

The circuit given permits a decrease of the static error of an MU of standard type (block 3) by a factor of 32.

If one of the MU's input voltages, say $x_1$, varies stepwise from −100 volts to +100 volts, the transient response time in the combination-type MU will equal the sum of the operation times of relays $Re_0$ to $Re_5$.

A disadvantage of this variation of the circuit for the rough portion is the voltage jump, small in magnitude and duration, at the MU's output at the moment when the relay contacts transfer. These jumps can be significantly decreased if polarized high-speed relays are used for the switching.

A relay variation can be made without splashes if, during the transient response (for large deviations), the exact portion is "roughened." Such a device was tested and gave good results although, it is true, the circuit of the MU was somewhat complicated thereby. In the $x_2$ channel, these MU's have frequency characteristics which are the same as those for ordinary operational amplifiers.

## Precision Multiplier Circuit with Reversible Selectors

The rough portion of an MU circuit can be constructed by means of a two-bank reversible stepping switch. The circuit of such an MU is shown in Fig. 4.

Two voltages, of opposite sign, are applied to the input of amplifier $A_1$: input voltage $x_1$ of the MU and the voltage from the brush $B_1$ of the switch (selector). This latter voltage is tapped off voltage divider $D_1$ via one of the commutator segments of the switch.

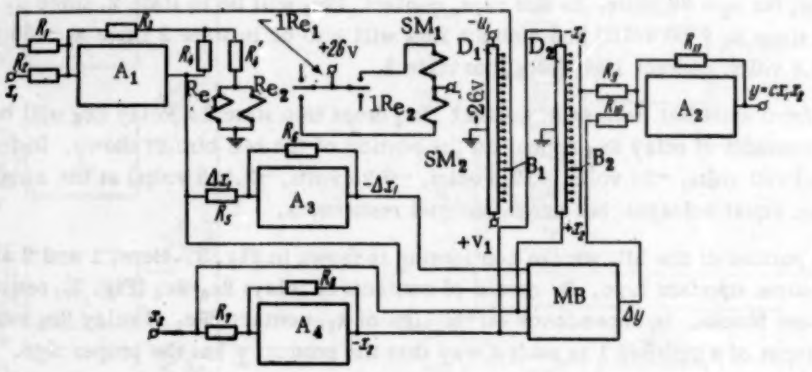


Fig. 4

The output voltage of amplifier $A_1$ ($\Delta x_1$) equals the difference of its input voltages amplified $k$ times. If voltage $\Delta x_1$ attains its limiting value (for example, 100 volts), two single-contact polarized relays $Re_1$ and $Re_2$ operate which, in dependence on the sign of $\Delta x_1$, lock in the winding of one of the electromagnets of the reversible switch (electromagnet $SM_1$ or $SM_2$). In this case, the brush $B_1$ of the switch is moved to a neighboring commutator segment, thanks to which voltage $\Delta x_1$ is decreased and becomes close to zero.

If, as a result of a change in $x_1$, voltage $\Delta x_1$ again attains the value of 100 volts, the switching described will again occur in the same order.

Simultaneously with $B_1$, brush $B_2$ steps along the commutator segments on the second bank of the switch. These segments are connected to voltage divider $D_2$. At the inputs of this divider are impressed voltages $+x_2$ and $-x_2$, equal in magnitude to the second factor in the MU. Two voltages are impressed on output amplifier $A_2$: one from brush $B_2$ and one from the MB block, which is an electronic multiplier of the usual type which multiplies voltages $\Delta x_1$ and $x_2$. To obtain normal operation of the MB, it is generally necessary to supply at its input both the positive and the negative values of the factors $\Delta x_1$ and $x_2$. In the circuit given, inversion of these voltages is effected by amplifiers $A_3$ and $A_4$.

As an example, let us consider a circuit in which the two-bank switch contains 40 commutator segments in each bank. We shall assume that the maximum value for the input and output voltages of the MU and MB equal 100 volts. We shall also assume that voltage $v_1 = 100$ volts. For this case, we shall determine the ratio of the impedances switched into the circuit.

For the selected values of the parameters, the voltage between neighboring switch commutator segments equals 5 volts. Consequently, switching must occur for values of $x_1$ equal to 5, 10, 15, . . . , 95 volts. Amplifier $A_1$ must have a gain of $k = 20$.

At the amplifier output we can set up a voltage divider consisting of resistors $R_4$ and $R_4'$ so that, for sufficiently large values of the MU's input impedance, resistor $R_3$ need not be too large. Amplifier $A_2$ must have equal gains for each of its inputs; the ratio $R_{11}/R_{10} = 1/20$, and ratio $R_{11}/R_9 = 1$.

As in the circuit considered earlier, in this circuit a small deviation of the relay operation voltage from the nominal value does not lead to an increased multiplication error if the scales of the input and output voltages of the ordinary MU slightly exceed 100 volts.

For the same end, the operation voltages of relays $Re_1$ and $Re_2$ are set to be slightly less than the scale

of the input voltage to the MB. With this, one also succeeds in eliminating the jumps in the MU'S output voltage and avoids the concomitant additional error in the MU, with the condition that the contacts between the commutator segments and brushes $B_1$ and $B_2$ are not disturbed. In the circuit given as an example, the error of the MU design is approximately 20 times lower than for an MU of the ordinary type.

In the given MU circuit one may, instead of the reversible selector switch, use the following system.

## An MU Circuit with a Relay-Type "Rough" Portion and a Thyrite "Exact" Portion

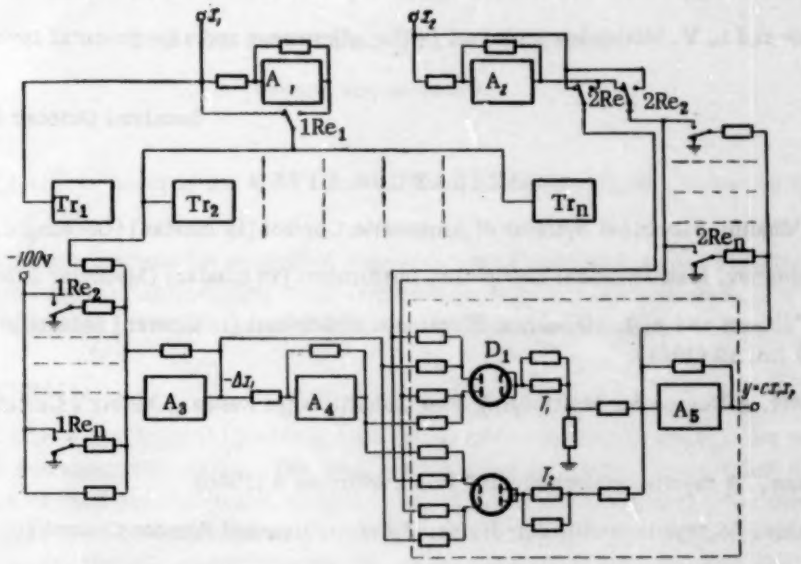


Fig. 5

The rough portion of the MU variant to be considered (Fig. 5) includes a row of symmetric triggers $Tr_1$-$Tr_n$, with different operating voltages. The number n of such triggers is determined by the admissible error of the MU. If, for example, the exact portion of the MU has an error of 0.5% of the output scale and the error of the combined MU must not exceed 0.05%, then the circuit must include 10 triggers with operating voltages of 0, 10, 20, . . . , 90 volts.

An electromagnetic relay is connected in the anode circuit of the vacuum tube in each of the triggers. These relays switch the impedances at the inputs of amplifiers $A_3$ and $A_5$. The exact portion of the MU (enclosed by the dotted line) includes two thyrites $T_1$ and $T_2$, diodes $D_1$ and $D_2$, and resistors. Its description is given in [5]. In order to decrease the number of relays and triggers, the inputs of the latter have only positive voltages applied to them.

This operation is performed by trigger $Tr_1$, which operates when $x_1 = 0$. For $x_1 < 0$, contact $1Re_1$ of this trigger's relay switches the input of the MU to the output of amplifier $A_1$, thus changing the sign of $x_1$. To assure that the output voltage has the correct sign, analogous switching is also carried out for input $x_2$ by means of contacts $2Re_1$ and $3Re_1$.

As in the previously considered circuits, the present circuit may, for a large number of problems, be considered practically inertialess for the $x_2$ channel. If polarized relays, for example, type RP, are used for switching, one may succeed in significantly increasing the rapidity of action of the $x_1$ channel as well. In processing, for example, a step function of arbitrary amplitude, the duration of the transient response will not exceed 3 to 5 milliseconds.

If it is necessary to obtain several products in which one of the factors is common ($x_1x_2$, $x_1x_3$, . . . ,$x_1x_n$), then the rough portion may be implemented as a common element for all the MU's. It is convenient to use this method to obtain, for example, a power series $a_1x + a_2x^2 + a_3x^3 + \ldots + a_nx^n$, with which one can frequently succeed, in computing devices, in reproducing a nonlinear function.

## SUMMARY

The circuits we have been considering for multipliers with "rough" and "exact" portions have greater accuracy and speed of action than do existing circuits. They are simple to adjust, and the elements comprising them are simple.

Experimental investigation of a number of circuits of multipliers constructed in accordance with the principles here considered were carried out as early as 1952, and gave good results. The static errors of such MU's did not exceed 0.01 to 0.02% of the output voltage scale.

Subsequently these components were manufactured as parts of nonlinear electronic models.



S. P. Onufriyuk and L. V. Medvedev took part in the adjustment and experimental investigation of the sample multipliers.

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# THE ANALYSIS AND SYNTHESIS OF CERTAIN DISCRETE CONTACTLESS CIRCUITS

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The article treats of the code circuits, based on diodes, which are used in the Strela computer.

Algebraic methods for analyzing and synthesizing such circuits are considered. The possible ways of constructing these circuits economically are discussed. Use is made of a logical symbolism close to the ordinary Boolean algebra.

## 1. Discrete Circuits

It is quite well known that logical (Boolean) algebra has been successfully used in the analysis and synthesis of circuits based on electromagnetic relays. The possibility of such an application was first discussed by V. I. Shestakov [1] for the so-called parallel-series circuits (P-circuits). Subsequent works contained the systematic application of the apparatus of Boolean algebra to the study of the most general circuits constructed of electromagnetic relays. Recently, the role of electromagnetic relays has been played, in many devices, by various contactless elements.*

In the present paper there is developed an apparatus which permits the analysis and synthesis of diode circuits to be carried out. This apparatus is a variant of the ordinary Boolean algebra, and not less simple than this latter.

First, we must define what we shall mean by a discrete circuit.

Let there be given some electrical circuit Z in which certain points are distinguished A, B, C, . . . , and let there exist for each of these points a succession of moments of time $t_1(A)$, $t_2(A)$, . . . , $t_1(B)$, $t_2(B)$, . . . , such that the electric potential at the given points at these moments of time assume only two values: $a(t_1)$ and $\overline{a}(t_1)$, $a(t_2)$ and $\overline{a}(t_2)$, . . . , $b(t_1)$ and $\overline{b}(t_1)$, etc. These points and moments of time are said to be characteristics of the given circuit.

In what follows, we shall denote one of the two values of potential as 1 and the other as 0. Instead of the term "potential" we shall frequently use the term "voltage."

Such a definition of a discrete circuit is somewhat idealized. What it actually comes down to is this: that for studying the operation of a host of circuits it suffices to know only that the voltages at certain points of the circuits at certain times are, or are not, greater, or less, than certain levels. For this, a knowledge of the voltages at other points and at other moments of time is nonessential. Thus, the behavior of a discrete circuit is determined by the voltage distribution at the characteristic points of the circuit at the characteristic moments of time.

We give here several examples.

1) A digital bus in a serial computer. Here, each bus transmits binary numbers in the form of a temporal sequence of pulses. The characteristic point here is any point of the bus, and the characteristic moments of time are determined by the common system of machine synchronization.

*The questions of synthesis and analysis of electronic circuits were discussed most completely in [2].

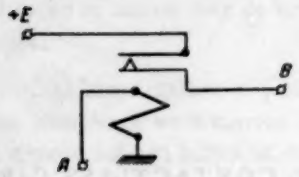



Fig. 1

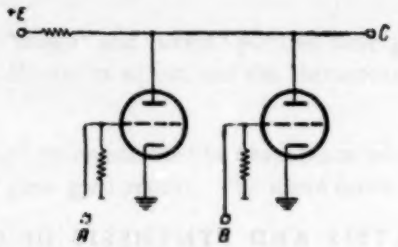



Fig. 2

For example, let the series of voltage values on the bus at times $t_1$, $t_2$, $t_3$, $t_4$, $t_5$ and $t_6$ equal (in our conventional symbolism) 1, 0, 1, 0, 0, 1. This means that the bus is transmitting the binary number 101001.

2) An electromagnetic relay (Fig. 1). The characteristic points A and B are shown on the sketch. The characteristic time corresponding to point B is defined by the condition that $t(B) \geq t(A) + \tau$, where $\tau$ is the contact make–break time.

Since the relay state is described in terms of make or break, it can be completely characterized by the voltage at point B being equal either to zero or to +E.

Unless something is said to the contrary, we shall assume in what follows that we are considering only static circuits, i.e., we shall assume that the state of the basic circuit Z does not depend on time.

A system of characteristic points of circuit Z will be called a complete system of inputs of Z if the voltages at all these points can be assigned independently and if the voltages at all the remaining characteristic points of the circuit are completely determined by the values of the voltages at the inputs. For example, in the circuit of Fig. 2, points A and B form a complete system of inputs.

For each characteristic point A of circuit Z one may form the proposition a: "The voltage at point A equals 1." The negation of this proposition is the proposition $\bar{a}$: "The voltage at point A equals 0."*

## 2. Structural and Characteristic Formulas

Here we may pose the general problem of analyzing relay-type circuits. Let there be given a circuit Z and a combination of voltages at the inputs. It is necessary to determine what the voltage will be at any previously designated characteristic point of the circuit.

We shall denote the characteristic points by the letters A, B, C, . . . , and will use the letters a, b, c, . . . , to denote the propositions: "the voltage at point A equals 1," "the voltage at point B equals 1" . . . Then, by the definition of input, every proposition of the form "the voltage at characteristic point F equals 1" may be written in the form of a formula of the propositional calculus in terms of the propositions a, b, c, . . . , for the corresponding inputs.

A system of formulas of the propositional calculus

$$\begin{aligned} f &= f(a, b, c, \ldots), \\ g &= g(a, b, c, \ldots), \\ h &= h(a, b, c, \ldots), \end{aligned} \tag{1}$$

written for every characteristic point of the circuit Z being considered will be called the system of characteristic formulas of circuit Z.

The problem of analysis of any discrete system can be thus formulated: given a circuit Z, to write its system of characteristic formulas.

*For a time-dependent circuit these propositions are generally meaningless, and should be replaced by propositions of the form a(t): "the voltage at point A at time t(A) equals 1."

The basic problem of synthesis of discrete systems can be formulated as follows: given a system of formulas of the propositional calculus which express the dependence of certain propositions f, g, h, . . . , on other propositions (1), to realize a circuit Z with inputs A, B, C, . . . , and characteristic points F, G, H, . . . , for which system (1) would be part of the system of characteristic formulas.

The example of a circuit built of electromagnetic relays shows that different circuits can be associated with one and the same set of characteristic formulas.

We are thus led to the necessity of characterizing each circuit by some structural formula in such a manner that different circuits will correspond to different structural formulas. For this, we shall consider as identic those circuits which are comprised of identical elements identically connected, i.e., circuits which can be symbolically represented by one and the same sketch.

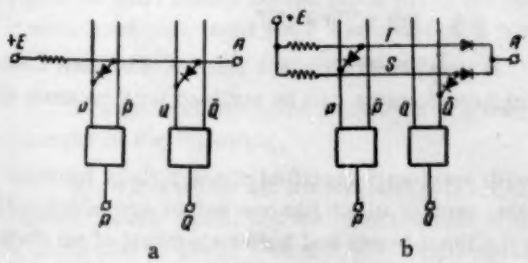
We shall assume here that we know how to set each discrete circuit, lying in some chosen class of circuits, in correspondence with a system of structural formulas in the sense used above. Then, the basic problem of analysis is formulated in this way: for a given system of structural formulas, to determine the system of characteristic formulas of the circuit. The basic problem of synthesis is formulated as: given a system of characteristic formulas, to find all the systems of structural formulas corresponding to it.

Of great interest are problems of the type: for a given system of characteristic formulas, to find structural formulas possessing certain extremal properties, for example, minimum number of elements in the circuit, minimum duration of the transient response when the voltage at the inputs is abruptly switched, optimum conductance. Extremal problems of this type are important in the applications of the theory, so that it is quite important to be able, by comparatively simple r.. .ns, to determine from a system of structural formulas what the properties of the corresponding circuit are.

It is shown in the works of V. I. Shestakov and of other authors that each circuit consisting of contact pairs connected in parallel or series (class P circuits, in the usual terminology) can be placed in correspondence with some formula of the propositional calculus. With this, the number of literal symbols entering into the circuit's structural formula equals the number of contact pairs. In this case, the characteristic formula of the network coincides with its structural formula. The essential result achieved here is that any formula of the propositional calculus can be realized as a structural (and, consequently, as a characteristic) formula of some relay-contact circuit of class P. This result showed the possibility, in theory, of synthesizing a circuit implementing any arbitrary logical function. From this there came the rules for transforming relay-contact circuits with the aim of decreasing the number of elements.

## 3. Diode Networks

In this work we shall consider networks whose basic elements are diodes. Circuits consisting of such diode nets can be used in the most general cases.* This follows from the result, obtained below, that any characteristic formula can be realized by means of such networks.




a b

Fig. 3

At this point we shall define what is to be understood by a network built of diode circuits. A network of this type consists of busses, connected by diode elements, and connecting voltage sources to them. In the sequel we shall assume that the busses have zero impedance and that the diodes are ideal. To each input terminal there corresponds a pair of input busses denoted by $(p, \bar{p})$. If there is a voltage on input p there will be a positive voltage +E on the first of these busses and zero voltage on the second. In the contrary case, there will be zero voltage on the first bus and +E on the second. In fact, both input busses are connected to the corresponding input terminal via a buffer element (switch), which may be implemented by a contact relay or an electronic tube. The input busses must satisfy the following requirements: 1) with no voltage the circuit is grounded; and, 2) no internal sources of voltage are connected to them.

*In particular, the arithmetic unit of the Strela computer is based on such circuits.

The remaining busses of the network are called working or output busses. The output terminals can be connected directly to any of the working busses.

To each point A on any of the working busses we may associate the proposition "A has voltage on it." Since the voltage on any bus can assume only two values, +E and 0, the proposition just stated (which we shall denote by the letter A), can be expressed by a formula of the propositional calculus in terms of propositions p, q, . . . , of the type: "input p has voltage on it."

The working busses are connected to the input busses by means of diodes which are always reversed with respect to the working busses.

Let us consider the examples of networks shown in Fig. 3.

In example a of Fig. 3, there will be high voltage on bus A only in the case when there is high voltage on both inputs P and Q. Indeed, in the other cases, if at least one of the P and Q busses had low potential on it, the voltage on the output would also be low due to the voltage drop across the resistance. Thus, the characteristic function of this network can be described by the equation $A = pq$.

In example b of Fig. 3, the voltage on the output will be high if there is either voltage on input P or no voltage on input Q. Indeed, in this case there will be voltage at the output only when there is voltage on one of the horizontal busses T or S. But current cannot flow between busses T and S and, consequently, bus T will have voltage on it only when there is high voltage on input P, and bus S will have voltage on it only when there is voltage on input $\overline{Q}$. Thus, the characteristic function of this network has the form:

$$A = p + \overline{q}.$$

Various examples show that the elementary logical functions are realized by means of diode networks. It is not difficult to convince oneself of the validity of the following assertion.

Theorem 1. For any formula of the propositional calculus* which contains propositions p, q, r, . . . , there exists a diode network each of whose inputs corresponds to one of the propositions p, q, r, such that its characteristic formula will be the initial formula.

Proof. Every formula of the propositional calculus can be represented in the form of a sum of products of the basic propositions p, q, r, . . . , or of their negations in such fashion that each product will contain either the proposition p (correspondingly for q, r, . . . ) or the proposition $\overline{p}$ and, moreover, will contain it just once.**

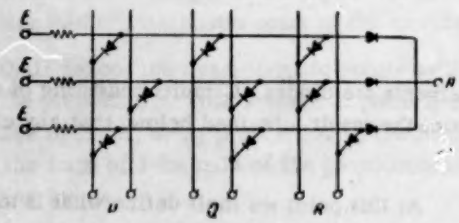


Fig. 4

We set up the network with inputs p, q, r, . . . that have working busses which realize the corresponding products, and we connect these busses to the outputs via diodes. It is easily seen that the characteristic formula of this network is precisely the corresponding sum of constituent elements. As an example, Fig. 4 shows the network corresponding to the formula

$$A = pqr + \overline{p}qr + p\overline{q}\overline{r}.$$

It is apparent from the proof of Theorem 1 that for networks of the type considered, with n inputs, any characteristic formula can be realized with no more than $(n + 1)2^{n-1}$ diodes.

In order to implement a network with several outputs, with previously specified characteristic formulas for each output, it suffices to implement the system of networks, each of which has one output and which realizes one of the proposed characteristic formulas, and then to joint the input busses and buffer elements of all these networks. From this follows Theorem 2. Let there be given n formulas which depend on the propositions p, q, r, .... Then there exists a network (diode net) whose outputs have the original formulas as their characteristic formulas.

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*The propositional calculus is sometimes called the algebra of logic, or Boolean algebra. The formulas of the propositional calculus are sometimes called Boolean functions.

**In other words, each formula of the propositional calculus is decomposed into sums of the constituent units (cf. [3]).

## 4. Propositional Calculus and Operators

The results of the previous section showed the possibility, in principle, of realizing any logical function by means of diode nets. These results, however, give no means of obtaining networks which are the most desirable, either in terms of the number of elements required or in terms of their physical properties.

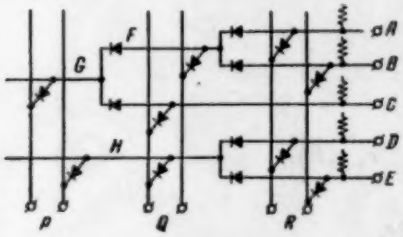


Fig. 5

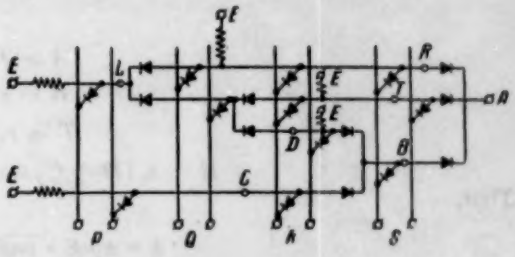


Fig. 6

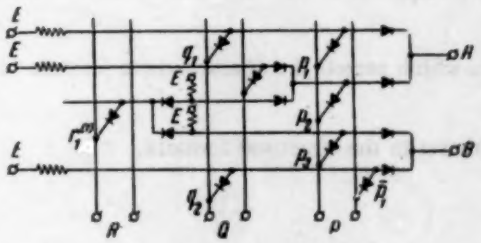


Fig. 7

For this purpose we shall now develop a formal apparatus which will permit the structural formulas of any diode net to be determined.

In contradistinction to the propositional calculus as ordinarily applied, we shall be dealing, in what follows, with quantities of a dual nature: with propositions and with operators. This bifurcation corresponds to the fact that the networks of the type considered consist of two forms of busses: input and working. For this, each input bus P breaks up into two input busses P and $\overline{P}$ in such fashion that if there is voltage on input P then bus P will have high voltage and $\overline{P}$ will have low voltage; in the contrary case, the voltage distribution on busses P and $\overline{P}$ will be reversed.

We introduce the following concept. Bus S will be called subordinate to bus T if, in the corresponding network it is possible to distinguish a sequence of busses, $R_1, R_2, \ldots, R_n$, such that the connection of bus T with bus $R_1$ is implemented by a diode which conducts current from bus T to bus $R_1$, busses $R_n$ and S are connected by a diode conducting current from $R_n$ to S; finally, any two busses $R_k$ and $R_{k+1}$ are connected by a diode conducting current in the direction from $R_k$ to $R_{k+1}$.

For example, in Fig. 5, bus G is subordinate to bus A and also to busses B and F, while bus H is subordinate to busses D and E.

To each working bus S there is associated the proposition "bus S has high voltage." We will denote this proposition by the letter s. To each input P there is associated the operator (proposition), "P has high voltage," which we shall denote by the letter p. In the sequel we shall find it convenient to assume that the operator p is associated with input bus P and operator $\overline{p}$ is associated with input bus $\overline{P}$. Each voltage source is associated with the tautologically true proposition E.

In a general form, the rules by which one can form the proposition S, corresponding to some working bus S,consist of the following.

1) We consider all working busses T, R, . . . , which are connected to bus S by diodes which conduct current toward bus S, and we form the sum of the propositions T + R + . . . . . In particular, if a voltage source is connected to bus S, then the tautology E should be added to the sum.

2) We next consider all input busses P, Q, R, each connected to one of the busses to which S is subordinate, and we apply successively the operator propositions p, q, . . . , to the proposition T + R + . . . . . As the result we obtain the expression

$$S = pq \ldots (T + R + \ldots).$$

3) Operators will bear upper and lower indices. The upper index on an operator $\underline{p}$ is one less than the number of diodes through which current must pass in order to flow between the working bus under consideration and input bus P. The lower index gives the ordinal number of the diode connected input bus P with the working bus. Each vertical bus will be numbered independently of the other.

Consider the example of Fig. 6. In accordance with the rules formulated, we have the sequence

$$A = R \vee T \vee B,$$
$$R = s_1 r_1 q_1 p_1^{(1)} E,$$
$$T = r_1 \bar{s}_1 q_1^{(1)} p_1^{(2)} E,$$
$$B = s_2 (D \vee C) = s_2 (\bar{q}_1^{(1)} p_1^{(1)} \bar{r}_1 \vee r_3 \bar{p}_1 E).$$

Thus,

$$A = srpqE + \bar{r}\bar{s}qE + s(\bar{q}p\bar{r} + rpE).$$

The operator calculus developed here allows each single-output network of the type considered to be unambiguously put in correspondence with a structural formula. One may also easily convince oneself that the converse is true: each formula of the operator calculus can be interpreted as the structural formula of some network (diode net) with a single output.

We are now in a position to formulate the following rules which permit the characteristic formula of a network to be derived from its structural formula.

1) By using the distributive law, multiply out any parentheses in the structural formula.

2) Delete every monomial which does not contain E.

3) Delete E from every monomial in which it appears.

4) Delete all indices from the operators.

5) The expression thus obtained is to be interpreted as a formula of the propositional calculus, the operators being considered as propositions.

Thus, the characteristic formula of the network shown in Fig. 6 has the form:

$$A = srqpE + \bar{r}\bar{s}\bar{q}pE + s(\bar{q}p\bar{r} + rpE) = srpq + s\bar{r}\bar{p}q + rps.$$

A rule can be stated for calculating the total number of diodes from the system of structural formulas. This rule reduces to the following:

1) Count the total number of terms $\nu$ in each structural formula.

2) For each operator $\underline{p}$, choose the maximum value of the lower index, and sum these values for all the operators. The result is denoted by $\mu$.

3) For each term in each structural formula, choose the maximum value of the upper index, and sum these values for all the terms. The result is denoted by $\lambda$.

The total number of diodes in the circuits equals the sum $\lambda + \mu + \nu$.

Example. Consider the system of structural formulas for some network:

$$A = p_1 q_1 E \vee p_2 (\bar{q}_1 E \vee r_1^{(1)} E),$$
$$B = \bar{p}_1 q_2 E \vee p_3 r_1^{(1)} E.$$

Let $\nu = 6$, $\mu = 8$, $\lambda = 2$.

Figure 7 shows the corresponding network.

## 5. Simplification of the Structural Formulas

We now consider transformations of the networks for which the system of characteristic formulas remains invariant.

## 4. Propositional Calculus and Operators

The results of the previous section showed the possibility, in principle, of realizing any logical function by means of diode nets. These results, however, give no means of obtaining networks which are the most desirable, either in terms of the number of elements required or in terms of their physical properties.

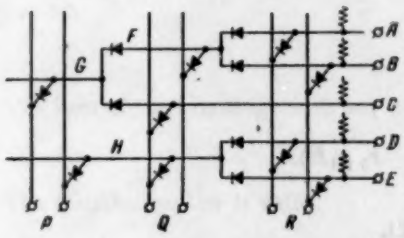


Fig. 5

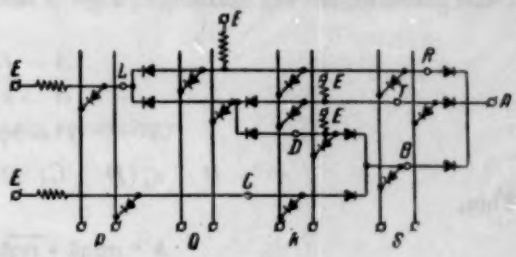


Fig. 6

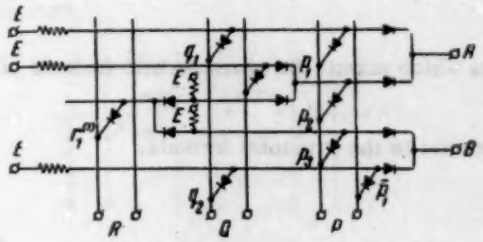


Fig. 7

For this purpose we shall now develop a formal apparatus which will permit the structural formulas of any diode net to be determined.

In contradistinction to the propositional calculus as ordinarily applied, we shall be dealing, in what follows, with quantities of a dual nature: with propositions and with operators. This bifurcation corresponds to the fact that the networks of the type considered consist of two forms of busses: input and working. For this, each input bus P breaks up into two input busses P and $\overline{P}$ in such fashion that if there is voltage on input P then bus P will have high voltage and $\overline{P}$ will have low voltage; in the contrary case, the voltage distribution on busses P and $\overline{P}$ will be reversed.

We introduce the following concept. Bus S will be called subordinate to bus T if, in the corresponding network it is possible to distinguish a sequence of busses, $R_1, R_2, \ldots, R_n$, such that the connection of bus T with bus $R_1$ is implemented by a diode which conducts current from bus T to bus $R_1$, busses $R_n$ and S are connected by a diode conducting current from $R_n$ to S; finally, any two busses $R_k$ and $R_{k+1}$ are connected by a diode conducting current in the direction from $R_k$ to $R_{k+1}$.

For example, in Fig. 5, bus G is subordinate to bus A and also to busses B and F, while bus H is subordinate to busses D and E.

To each working bus S there is associated the proposition "bus S has high voltage." We will denote this proposition by the letter s. To each input P there is associated the operator (proposition), "P has high voltage," which we shall denote by the letter p. In the sequel we shall find it convenient to assume that the operator p is associated with input bus P and operator $\overline{p}$ is associated with input bus $\overline{P}$. Each voltage source is associated with the tautologically true proposition E.

In a general form, the rules by which one can form the proposition S, corresponding to some working bus S,consist of the following.

1) We consider all working busses T, R, . . . , which are connected to bus S by diodes which conduct current toward bus S, and we form the sum of the propositions T + R + . . . . . In particular, if a voltage source is connected to bus S, then the tautology E should be added to the sum.

2) We next consider all input busses P, Q, R, each connected to one of the busses to which S is subordinate, and we apply successively the operator propositions p, q, . . . , to the proposition T + R + . . . . . As the result we obtain the expression

$$S = pq \ldots (T + R + \ldots).$$

3) Operators will bear upper and lower indices. The upper index on an operator $\underline{p}$ is one less than the number of diodes through which current must pass in order to flow between the working bus under consideration and input bus P. The lower index gives the ordinal number of the diode connected input bus P with the working bus. Each vertical bus will be numbered independently of the other.

Consider the example of Fig. 6. In accordance with the rules formulated, we have the sequence

$$A = R \dotplus T \dotplus B.$$
$$R = s_1 r_1 q_1 p_1^{(1)} E,$$
$$T = r_1 s_1 q_1^{(1)} p_1^{(2)} E,$$
$$B = s_2 (D \dotplus C) = s_2 (\bar{q}_1^{(1)} p_1^{(1)} \bar{r}_1 \dotplus r_3 \bar{p}_1 E).$$

Thus,

$$A = srpqE + \bar{r}\bar{s}\bar{q}E + s(\bar{q}p\bar{r} + rpE).$$

The operator calculus developed here allows each single-output network of the type considered to be unambiguously put in correspondence with a structural formula. One may also easily convince oneself that the converse is true: each formula of the operator calculus can be interpreted as the structural formula of some network (diode net) with a single output.

We are now in a position to formulate the following rules which permit the characteristic formula of a network to be derived from its structural formula.

1) By using the distributive law, multiply out any parentheses in the structural formula.

2) Delete every monomial which does not contain E.

3) Delete E from every monomial in which it appears.

4) Delete all indices from the operators.

5) The expression thus obtained is to be interpreted as a formula of the propositional calculus, the operators being considered as propositions.

Thus, the characteristic formula of the network shown in Fig. 6 has the form:

$$A = srqpE + \bar{r}s\bar{q}pE + s(\bar{q}p\bar{r} + rpE) = srpq + s\bar{r}pq + rps.$$

A rule can be stated for calculating the total number of diodes from the system of structural formulas. This rule reduces to the following:

1) Count the total number of terms $\nu$ in each structural formula.

2) For each operator $\underline{p}$, choose the maximum value of the lower index, and sum these values for all the operators. The result is denoted by $\mu$.

3) For each term in each structural formula, choose the maximum value of the upper index, and sum these values for all the terms. The result is denoted by $\lambda$.

The total number of diodes in the circuits equals the sum $\lambda + \mu + \nu$.

Example. Consider the system of structural formulas for some network:

$$A = p_1 q_1 E \dotplus p_2 (\bar{q}_1 E \dotplus r_1^{(1)} E),$$
$$B = \bar{p}_1 q_2 E \dotplus p_3 r_1^{(1)} E.$$

Let $\nu = 6$, $\mu = 8$, $\lambda = 2$.

Figure 7 shows the corresponding network.

## 5. Simplification of the Structural Formulas

We now consider transformations of the networks for which the system of characteristic formulas remains invariant.

For this, we formulate the following rules for operating with propositions and operators.

1) Addition of propositions is commutative, so that

$$A + B = B + A.$$

2) Propositions can be multiplied by operators, the result being a proposition and the following relationship being valid:

$$pqA = qpA.$$

3) The sum of the tautology with any proposition is again a tautology:

$$E + A = E.$$

4) The distributive law is valid:

$$p(A + B) = pA + pB.$$

It is clear that with such transformations of the structural formulas there is no change in the characteristic formulas, and that with each such transformation one obtains a structural formula which corresponds to some actual network.

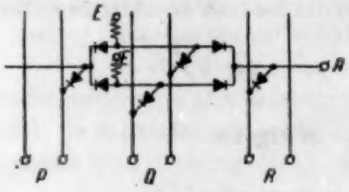


Fig. 8

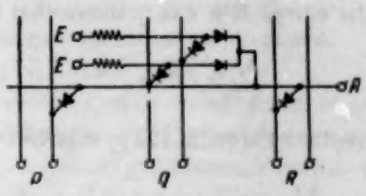


Fig. 9

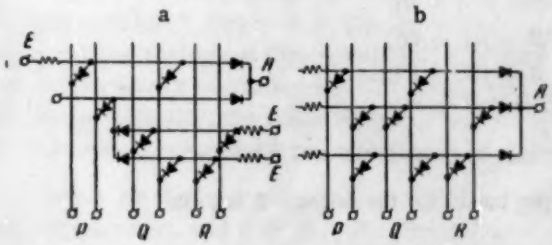


Fig 10

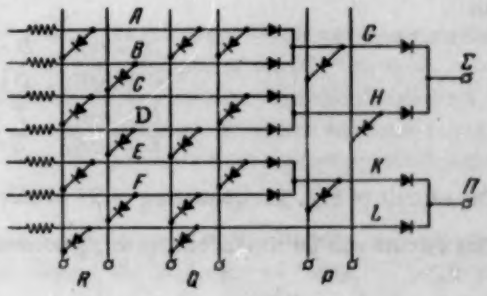


Fig. 11

Let us now consider in more detail transformation of networks with one output. We note immediately that in this case the logical apparatus can be simplified by dropping the indices on the operator propositions in the formulas. In fact, the necessity for having upper indices was due only to the fact that some input bus could have been connected to two working busses by means of one diode. In such a case it was necessary that these working busses be separated by diodes, and the purpose of the upper index was just to allow these separating diodes to be taken into account.

As an example, we take Fig. 8. Such a method for connecting busses turns out to be possible when there is a common factor in several terms. However, it is clear from this same Fig. 8 that by transforming this network to the form shown in Fig. 9 we can save two diodes by eliminating the separating diodes.

The example just considered shows that in the general case it is possible to avoid the use of isolating (separating) diodes. But in this case it is no longer necessary to retain either the upper or the lower indices, since each diode can be used only for connecting one factor in the structural formulas to an input bus. Here, the rule for computing the number of diodes realizing the given structural formulas can be formulated as follows: the total number of diodes is equal to the number of terms in the structural formulas plus the number of all the operators entering into these formulas. With this, each operator is counted as many times as it enters into the formulas.

Example. The number of diodes realizing the structural formula:

$$A = p\bar{q}E + \bar{p}(q\bar{r}E + \bar{q}rE),$$

equals 4 + 7 = 11.

This formula can be transformed to the form

$$A = p\bar{q}E + \bar{p}q\bar{r}E + \bar{p}\bar{q}rE.$$

In this case, the number of diodes equals 3 + 8 = 11. The corresponding networks are shown in Fig. 10, where Fig. 10a corresponds to the first of these formulas and Fig. 10b corresponds to the second.

Thus, simplification of structural formulas should be effected by factoring out common factors in the terms. In so doing, however, the total number of terms should not be too greatly increased, since such an increase can lead to a deterioration of the frequency characteristics of the circuit, to the appearance of false circuits, and to disruptions of circuit operations due to the finite magnitude of the diodes' back resistance.

The same rules for simplification remain valid also for circuits with several outputs. In addition, however, certain special simplifications are possible if several outputs have common factors or terms.

As an example, consider a one-bit binary adder with three inputs P, Q and R, and two outputs: $\Sigma$ is the sum and $\Pi$ is the carry. It is easily shown that the characteristic formulas for such an adder have the forms:

$$\sum = \bar{p}\bar{q}r + \bar{p}q\bar{r} + p\bar{q}\bar{r} + pqr, \qquad \prod = pqr + pq\bar{r} + p\bar{q}r + \bar{p}qr. \tag{2}$$

We can replace Formulas (2) by equivalent formulas of the logic of algebra

$$\sum = p(\bar{q}\bar{r} + qr) + \bar{p}(\bar{q}r + q\bar{r}), \qquad \prod = rq + p(r\bar{q} + \bar{r}q). \tag{3}$$

By introducing the corresponding propositions one can, from Formulas (3), construct the system of structural formulas:

$$\begin{aligned} &A = rqE, \quad B = \bar{r}\bar{q}E, \quad G = p(A + B),\\ &C = r\bar{q}E, \quad D = \bar{r}qE, \quad H = \bar{p}(C + D),\\ &E = \bar{r}\bar{q}E, \quad F = r\bar{q}E, \quad K = p(E + F),\\ &L = rqE, \quad \Sigma = G + H, \ \Pi = H + L. \end{aligned}$$

The circuit of Fig. 11, consisting of 27 diodes, corresponds to these structural formulas.

This circuit can be simplified by using common working busses for the outputs $\Sigma$ and $\Pi$.

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# CHAIN MATRICES AND SUBSTITUTION CIRCUITS FOR ELECTROMAGNETIC-TYPE ELECTROMECHANICAL TRANSFORMERS

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Different variations of four-terminal substitution circuits for electromagnetic-type electromechanical transformers are considered. The possible methods of estimating the influence of "negative rigidity" on transformer operation are discussed.

Electromagnetic-type electromechanical transformers are widely used in the apparatus of automation and remote control. In particular, polarized transformers with linear characteristics are used in tuned relays, electromechanical filters [1,2,3], tuning-fork frequency regulators [4,5], vibration gyroscopes, vibration transducers, etc.

We shall consider the processes occurring both in the electrical and in the mechanical portions of the transformer, since the transformer is most effectively considered as an electromagnetic four-terminal network, described by the corresponding matrices [2, 6-9].

In [8] there was proposed a matrix system for different types of transformers which was based on the analogy of "force-voltage." However, it was shown in [7] that in order for the principle of duality* to hold, it is necessary to use the analogy of "force-current" in setting up the matrices of the electromagnetic transformer. Moreover, in the matrix system used in [8], phenomena attendent on the transformer processes were not taken into account, specifically, the occurrence of a negative component in the expression for the mechanical rigidity of the electromagnetic transformer (described, for example, in [1]).

Below we shall compare various methods of matrix and schematic presentation of polarized electromagnetic transformers, and shall give the possible methods for estimating the influence of "negative rigidity" on their operation.

## 1. Chain Matrices of an Electromagnetic Transformer

Figure 1 gives the schematic representation of an electromechanical four-terminal network which corresponds to an electromagnetic transformer with a movable core. The directions chosen for the voltage U, current I, force F and velocity V are nominal. Four-terminal network a transforms electrical energy into mechanical energy, while network b transforms mechanical energy into electrical. A series (chain-like) connection of such four-terminal networks is possible, both on the electrical and on the mechanical side (for example, in electromechanical frequency filters).

The system of equations which describes the processes in an ideal electromagnetic polarized transformer working with small sinusoidal oscillations (account not being taken of magnetic leakage and losses in the electrical portion) has the following form:

$$\begin{aligned} \dot{U} &= j\omega L_b' \dot{I} + \beta \dot{V}, \\ \dot{F} &= \beta \dot{I} - Z_m \dot{V}. \end{aligned} \tag{1}$$

*It is necessary that the duality principle hold in order that the electromechanical transformer might be replaced by the purely electrical passive circuit equivalent in form [7].

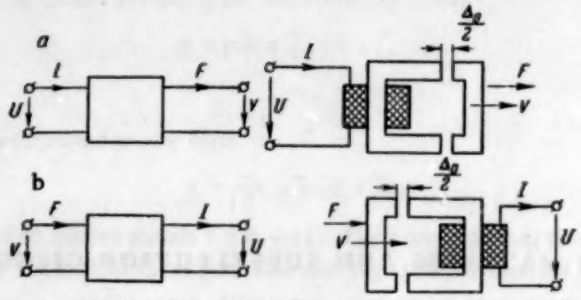


Fig. 1

Here, $Z_m$ is the mechanical resistance of the moving portion of the transformer with the electrical circuit open, $\beta$ is the coefficient of the electromechanical connection, $L_b'$ is the inductance of the braked transformer (cf. Appendix).

The force F engendered by the total magnetic flux $\Phi$ of the transformer is defined by the expression

$$F = -\frac{\Phi^2}{2\mu_0 S} = -\frac{(\Phi_0 + \varphi)^2}{2\mu_0 S}, \tag{2}$$

where $\Phi_0$ is the constant polarizing magnetic flux, $\varphi$ is the variable magnetic flux generated by the coil current i (ordinarily, $\Phi_0 \gg \varphi$), $\mu_0$ is the magnetic permeability, and S is the cross section area of the effective air gap.

With no alternating current in the transformer coil, the change in force can be expressed in the form (cf. Appendix):

$$dF = +\frac{\Phi_0^2}{\mu_0 S \Delta_0} d\delta, \tag{3}$$

where $\Delta_0$ is the constant, and $\delta$ the variable, component of the air gap width.

One of the effects of the polarizing field is the engendering of an additional mechanical force (3), acting upon the armature in the direction away from its equilibrium position. This effect leads to the appearance of a negative component of mechanical rigidity of the transformer, this component equalling

$$-\frac{dF}{d\delta} = -\frac{\Phi_0^2}{\mu_0 S \Delta_0}. \tag{4}$$

It follows that the mechanical impedance will consist of two parts:

$$Z_m = Z_m' + Z_m'' = Z_m' - \frac{1}{j\omega}\frac{dF}{d\delta} = Z_m' - \frac{\beta^2}{j\omega L_b'}, \tag{5}$$

where $Z_m'$ is the inherent mechanical impedance, and $Z_m''$ is the impedance of the "negative rigidity."

After substitution of expression (5) for $Z_m$ in (1) and some regrouping of terms, we obtain a new system of equations which, in matrix notation, has the following form:

$$\begin{Vmatrix} \dot{U} \\ \dot{I} \end{Vmatrix} = \begin{Vmatrix} j\omega L_b' \frac{Z_m'}{\beta} & j\omega L_b' \frac{1}{\beta} \\ \frac{Z_m'}{\beta} - \frac{\beta}{j\omega L_b'} & \frac{1}{\beta} \end{Vmatrix} \times \begin{Vmatrix} \dot{V} \\ \dot{F} \end{Vmatrix} \tag{6}$$

The first factor in the right member of (6) is the chain matrix of the transformer. The rationale for this name is that, for a series connection of transformers (in a "chain"), the chain matrix of the whole series equals the product of the chain matrices of the individual transformers.

## 2. Circuit for a Substitution Transformer

In substitution circuits it is effective to connect the corresponding electrical and mechanical elements on the electrical and mechanical sides of the transformer. Between these elements is to be found the specific four-terminal network which relates the electrical and mechanical quantities.

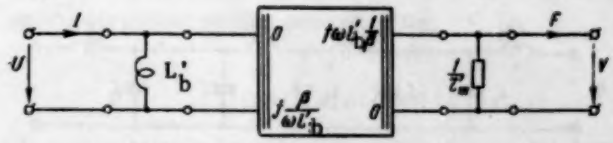

Fig. 2. Substitution circuit of a transformer with a four-terminal network for an electromechanical connection depending on frequency.

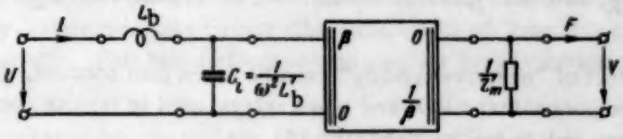

Fig. 3. Substitution circuit of a transformer with a four-terminal network for an electrical connection not depending on frequency.

The substitution circuit can be obtained by decomposing the chain matrix of (6) into its constituent matrices. One possible decomposition gives the following matrix product:

$$\begin{Vmatrix} 1 & 0 \\ \frac{1}{j\omega L'_b} & 1 \end{Vmatrix} \times \begin{Vmatrix} 0 & j\omega L'_b \frac{1}{\beta} \\ j\frac{\beta}{\omega L'_b} & 0 \end{Vmatrix} \times \begin{Vmatrix} 1 & 0 \\ Z'_m & 1 \end{Vmatrix} \tag{7}$$

According to the theory of four-terminal networks, the circuit of Fig. 2 corresponds to expression (7). The circuit given has the disadvantage that the transformation of the mechanical element to an electrical quantity, and vice versa, depends on frequency (since the factor $\omega$ occurs in the matrix of the four-terminal network of the electromechanical connection).

Another method of decomposing the chain matrix in (6) gives the following matrix product:

$$\begin{Vmatrix} 1 & j\omega L'_b \\ 0 & 1 \end{Vmatrix} \times \begin{Vmatrix} 1 & 0 \\ j\omega \frac{1}{\omega^2 L'_b} & 1 \end{Vmatrix} \times \begin{Vmatrix} \beta & 0 \\ 0 & \frac{1}{\beta} \end{Vmatrix} \times \begin{Vmatrix} 1 & 0 \\ Z'_m & 1 \end{Vmatrix} \tag{8}$$

Figure 3 corresponds to expression (8). Its advantage is that the mechanical quantity is transformed to an electrical quantity, and vice versa, independently of frequency, since the matrix of the four-terminal network of the electromechanical connection does not contain $\omega$. Moreover, the circuit in Fig. 3 allows the effect of "negative rigidity" to be estimated graphically.

The quantity which is the reciprocal of the rigidity K of the mechanical system is transformed, on the electrical side, to an inductance

$$L_K = \frac{\beta^2}{K} = \beta \frac{w\Phi_0}{\Delta_0} \frac{1}{K} = \beta \frac{1}{K} \frac{I_0}{\Delta_0} L'_b, \tag{9}$$

where $\underline{w}$ is the number of turns in the transformer's coil and $I_0$ is the current necessary to give rise to the polarizing flux $\Phi_0$.

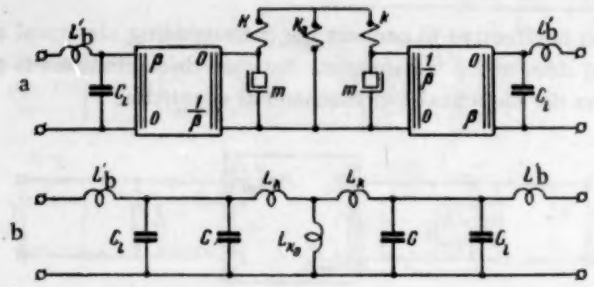


Fig. 4

If K is sufficiently large, then $L'_b \gg L_K$. Consequently, the admittance $j\omega C_L = j/\omega L'_b$, in comparison with the admittance $1/j\omega L_K$, can be neglected; in this case, the capacitance $C_L$ will not appear in the circuit of Fig. 3.

In many cases the effect of "negative rigidity" must be taken into account. In particular, it can be used for the fine tuning of electromechanical filters and tuned relays, used in remote control frequency devices [1,3] and in the adjusting of tuning-fork frequency regulators [5], etc.

As an example, we may take the substitution circuit for an electromechanical band-pass filter with flexibly connected vibrators which was developed for frequency telecontrol devices. Fig. 4a shows the equivalent electromechanical circuit, and Fig. 4b shows the electrical circuit which replaces such a filter. In the given case, the circuit parameters had the following values:

$$\beta = 30\,\frac{\text{v·sec}}{\text{м}}, \quad C = \frac{m}{\beta^2} = \frac{0.53\times10^{-3}\ \text{kg·m}^2}{30^2\ \frac{\text{v}^2\text{sec}^2}{\text{м}^2}} = 0.59\times10^{-6}\ f,$$

$$L_K = \frac{\beta^2}{K} = \frac{30^2}{290\times10^2}\,\frac{\text{v}^2\text{sec}^2}{\text{м}^2}\,\frac{\text{м}}{\text{N}} = 0.031\ \text{h},$$

$$L_{K_0} = \frac{\beta^2}{K_0} = \frac{30^2}{320\times10^4} = 0.28\times10^{-3}\ \text{h} \quad L'_b = 0{,}9\ \text{h}$$

(m is the equivalent mass of the vibrator, N = newton).

With this, the natural frequency of the tuned vibrator, corresponding to the upper boundary of the filter's pass band, equals

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{L_K C}} \approx 1180\ \text{cps}.$$

With the vibrators in the magnetic field of the transformer filter with a small air gap (0.2 mm), the effect of "negative rigidity" (element $C_L$ in the circuits of Fig. 4) appeared. The magnitude of $C_L$ in the case given equals

$$C_L = \frac{1}{\omega^2 L'_b} \approx 0\ 02\times10^{-6}\ f.$$

Due to the effect of $C_L$ the natural frequency of the vibrators must be reduced to the value

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{L_K(C+C_L)}} \approx 1160\ \text{cps}$$

which actually occurs.

When account is taken of the voltage drop across the coil's ohmic impedance and of the inductance leakage, as well as the active component of the input current, related to the loss in the iron (core), the transformer substitution circuit becomes somewhat more complicated (Fig. 5). Here are appended the element $L''_b$, the inductance leakage, the winding impedance $R_b$ and also the loss impedance $R_i$ in the iron. All the previous discussion applies also to this circuit, starting with terminals 1 and 2.

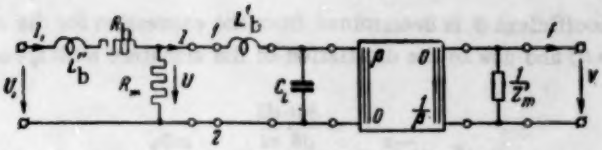

Fig. 5. Transformer substitution circuit with dissipation in the electrical portion accounted for.

## CONCLUSIONS

Chain matrices and substitution circuits for electromagnetic circuits can have different forms, depending on the type of analogy employed and on the method of decomposing the matrices. The duality principle holds if the analogy "force-current" is used. For this, the most convenient substitution circuit is that in which the matrix of the four-terminal electromechanical connection does not contain a factor of frequency. In this circuit, the transformation of electrical element to mechanical element, and the converse transformation, occurs independently of frequency. Moreover, this circuit allows the effect of "negative rigidity" on transformer operation to be estimated graphically. This latter effect can be used for the regulation of frequency tuning in resonant systems.

The author wishes to express his gratitude to P. I. Zubkov for his valuable comments on the questions considered here.

## Appendix

1. The total differential force acting on the moving portion of the electromagnetic transformer equals

$$dF = \frac{\partial F}{\partial i}di + \frac{\partial F}{\partial \delta}d\delta = \frac{\partial F}{\partial \Phi}\frac{\partial \Phi}{\partial i}di + \frac{\partial F}{\partial \Phi}\frac{\partial \Phi}{\partial \delta}d\delta.$$

The first term of this expression defines the variable force created by the coil's alternating current $i$. The value of the second term is found from the definition of its factors for $i = 0$:

$$(F)_{i=0} = -\frac{\Phi_0^2}{2\mu_0 S}, \quad \left(\frac{\partial F}{\partial \Phi}\right)_{i=0} = -\frac{\Phi_0}{\mu_0 S}.$$

From the law of total current

$$\Phi = \frac{w(I_0 + i)\mu_0 S}{\Delta_0 + \delta}$$

it follows that (for $\Delta_0 \gg \delta$)

$$\left(\frac{\partial \Phi}{\partial \delta}\right)_{i=0} = -\frac{wI_0\mu_0 S}{\Delta_0^2} = -\frac{\Phi_0}{\Delta_0}.$$

If we take the above expressions into account, we obtain

$$(dF)_{i=0} = \left(-\frac{\Phi_0}{\mu_0 S}\right)\left(-\frac{\Phi_0}{\Delta_0}\right)d\delta = +\frac{\Phi_0^2}{\mu_0 S \Delta_0}d\delta.$$

2. $L_b'$ is the coefficient of proportionality between the increment of effective flux linkage and the increment of current. Since, for a braked transformer, we have the relationships

$$\Delta\Phi = \Phi - \Phi_0 = \varphi, \quad \varphi = \frac{wi\mu_0 S}{\Delta_0 + \delta} \quad \text{then} \quad L_b' = \frac{w\varphi}{i} = \frac{w^2\mu_0 S}{\Delta_0}.$$

Thus, $L_b'$ is the inductance of the braked transformer without account being taken of the flux leakage.

3. The value of the coefficient β is determined from the expression for the emf e directed to the transformer's opened coil (for i = 0) and due to the oscillation of the armature with speed v:

$$\beta = \frac{-e}{v} = \frac{w \frac{\partial \Phi}{\partial \delta} \frac{d\delta}{dt}}{\frac{d\delta}{dt}} = -\frac{w\Phi_0}{\Delta_0}.$$

This expression can also be obtained from the dependence between current and force for a braked transformer:

$$dF = \frac{\partial F}{\partial i} di = \frac{\partial F}{\partial \Phi} \frac{\partial \Phi}{\partial i} di = -\frac{\Phi_0}{\mu_0 S} \frac{\mu_0 S w}{\Delta_0} di = -\frac{w\Phi_0}{\Delta_0} di,$$

so that

$$\beta = \frac{dF}{di} = -\frac{w\Phi_0}{\Delta_0}.$$

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# ON THE POSSIBILITY OF CERTAIN TYPES OF OSCILLATIONS IN SAMPLED-DATA CONTROL SYSTEMS

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## 1. Posing of the Problem

We shall consider an automatic control system containing a linear portion and a pulse (sampled-data) element (Fig. 1). As shown in Fig. 2, the system is characterized by the repetition period $T_s$, the pulse amplitude $k\,\mathrm{sign}\,x(jT_s)$ and length $T_j = g|x(jT_s)|$.*

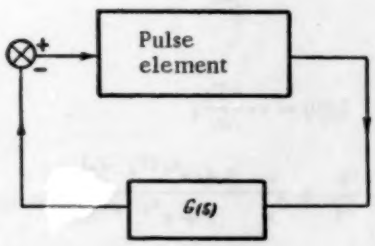


Fig. 1

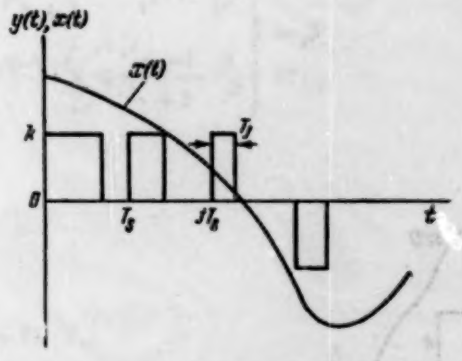


Fig. 2

We shall assume that the transfer function of the linear portion of the system has the form:

$$G(s) = \frac{1}{sQ(s)} = \frac{1}{s(s-\lambda_2)(s-\lambda_3)\ldots(s-\lambda_n)},$$

where $\mathrm{Re}\,\lambda_i < 0$ and the gain is taken equal to unity; in other cases it may be taken equal to a constant $k$. We now consider the possibility of there being symmetric oscillations with period $2T_s$. The presence of the integrating term makes the existence of asymmetric auto-oscillations impossible. We choose the origin ($t = 0$) in such fashion that the input is positive (Fig. 3), and then

$$T_1 = gx(0). \tag{1}$$

It suffices to consider the process during the half-period $T_s$.

## 2. Periodic Solution

We first transform the equation of the linear portion

$$pQpx = -y, \tag{2}$$

*If $g|x(jT_s)| > T_s$, then $T_j = T_s$.

where $p = d/dt$, to the normal form [1]:

$$\frac{d\xi_1}{dt} = y,$$
$$\frac{d\xi_i}{dt} = \lambda_i \xi_i + y \qquad (i = 2,3,\ldots,n). \tag{3}$$

Then

$$x = -\frac{1}{Q(0)}\,\xi_1 - \sum_{n-2}^{n} \frac{1}{\lambda_i Q'(\lambda_i)}\,\xi_i\,. \tag{4}$$

Figure 3 shows the form of y(t) when there are oscillations. With this,

$$y = k \text{ for } 0 < t < T_1,\ y = 0 \text{ for } T_1 < t < T_s.$$

Moreover, $\xi_i$ is continuous and $\xi_i(0) = -\xi_i(T_s)$.

Taking these conditions into account, we present the solution of Eq. (3) in the form

$$\xi_1 = \begin{cases} kt - \dfrac{kT_1}{2}, & 0 < t < T_1; \\ \dfrac{kT_1}{2} = \xi_1(T_1), & T_1 < t < T_s, \end{cases}$$

$$\xi_i = \begin{cases} \dfrac{-k}{\lambda_i} + \dfrac{k}{\lambda_i}\,\dfrac{1 + e^{\lambda_i (T_s - T_1)}}{1 + e^{\lambda_i T_s}}\,e^{\lambda_i t}, & 0 < t < T_1; \\ \dfrac{k}{\lambda_i}\,\dfrac{1 - e^{\lambda_i T_1}}{1 + e^{\lambda_i T_s}}\,e^{\lambda_i (t - T_1)} = \xi_i(T_1) e^{\lambda_i (t - T_1)}, & T_1 < t < T_s, \end{cases} \tag{5}$$

where

$$\xi_1(0) = -\frac{kT_1}{2},$$

$$\xi_i(0) = -\frac{k}{\lambda_i} + \frac{k}{\lambda_i}\,\frac{1 + e^{\lambda_i (T_s - T_1)}}{1 + e^{\lambda_i T_s}} = -\frac{k}{\lambda_i}\,\frac{1 - e^{-\lambda_i T_1}}{1 + e^{-\lambda_i T_s}}$$

and

$$x(0) = \frac{kT_1}{2Q(0)} + \sum_{i-2}^{n} \frac{k}{\lambda_i^2 Q'(\lambda_i)}\,\frac{1 - e^{-\lambda_i T_1}}{1 + e^{-\lambda_i T_s}}\,. \tag{6}$$

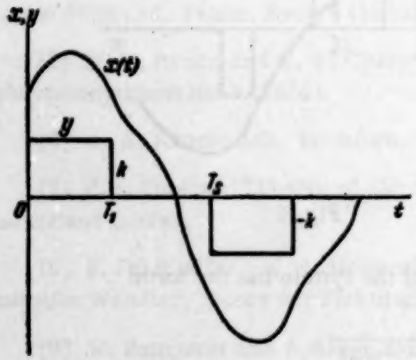


Fig. 3

From (1) and (6) we obtain

$$\left(\frac{1}{2Q(0)} - \frac{1}{gk}\right) T_1 = -\sum_{i-2}^{n} \frac{1}{\lambda_i^2 Q'(\lambda_i)}\,\frac{1 - e^{-\lambda_i T_1}}{1 + e^{-\lambda_i T_s}}\,. \tag{7}$$

If Eq. (7) has a real and positive solution $T_1 > 0$, then this means that the form of oscillation considered is possible, while the contrary case means that such an oscillation is impossible.

We obtain the complete solution by substituting $T_1$ in (5). It is possible to estimate the relative magnitude of $T_1$ since $x(0) = T_1/g$. For $T_1 > T_s$ one should set $T_1 = T_s$. With this,

$$x(\theta) = \frac{kT_s}{2Q(0)} + \sum_{i-2}^{n} \frac{k}{\lambda_i^2 Q'(\lambda_i)}\,\text{th}\,\frac{\lambda_i T_s}{2}\,.$$

## 3. Stability of the Oscillations

We shall assume that there is an oscillation. We now consider a small disturbance $\xi_i$ of the variable $\underline{x}$. If this perturbation tends to zero as $t \to \infty$, then the oscillation is stable ("in the small"). If we denote the periodic solution for $\xi_i$ by $\xi_i^*$ and the perburbation by $\delta\xi_i$, we then have

$$\xi_1 = \xi_1^* + \delta\xi_1, \qquad \xi_i = \xi_i^* + \delta\xi_i \qquad (i = 2, \ldots, n). \tag{8}$$

Since $\xi_1$ and $\xi_i$ are solutions of Eq. (3), the equations in the variations will take the form:

$$\frac{d\delta\xi_1}{dt} = 0, \quad \frac{d\delta\xi_i}{dt} = \lambda_i \delta\xi_i \qquad (i = 2, \ldots, n), \tag{9}$$

$$\delta\xi_1 = \delta\xi_1(0) = \text{const},$$

$$\delta\xi_i = \delta\xi_i(0)\, e^{\lambda_i t} \qquad (i = 2, \ldots, n). \tag{10}$$

Equations (10) are valid in the interval $0 < t < T_1'$, where

$$T_1' = gx(0) = -\frac{g}{Q(0)}\xi_1(0) - \sum_{i=2}^{n} \frac{g}{\lambda_i Q'(\lambda_i)}\xi_i(0) = \sum_{i=1}^{n} G_i \xi_i(0),$$

where

$$G_1 = -\frac{g}{Q(0)}, \quad G_i = -\frac{g}{\lambda_i Q'(\lambda_i)} \qquad (i = 2, \ldots, n), \tag{11}$$

or

$$T_1' = \sum_{i=1}^{n} G_i[\xi_i^*(0) + \delta\xi_i(0)] = \sum_{i=1}^{n} G_i \xi_i^*(0) + \sum_{i=1}^{n} G_i \delta\xi_i(0) = T_1 + \delta T_{(1)},$$

where

$$\delta T_{(1)} = \sum_{i=1}^{n} G_i \delta\xi_i(0). \tag{12}$$

For $t = T_1'$

$$\xi_i(T_1') = \xi_i^*(T_1') + \delta\xi_i(T_1').$$

But in the interval $T_1' < t < T_2$, $y = 0$ and

$$\xi_1 = \xi_1^*(T_1) + \delta\xi_1(T_1') = \xi_1^*(T_1') + \delta\xi_1(0).$$

$$\xi_i = [\xi_i^*(T_1') + \delta\xi_i(T_1')]\, e^{\lambda_i (t - T_1')} = [\xi_i^*(T_1') + \delta\xi_i(0)\, e^{\lambda_i T_1'}]\, e^{\lambda_i (t - T_1')} \qquad (i = 2, \ldots, n)$$

For $t = T_2$ we get

$$\xi_1(T_2) = \xi_1^*(T_1') + \delta\xi_1(0),$$

$$\xi_i(T_2) = [\xi_i^*(T_1') + \delta\xi_i(0)\, e^{\lambda_i T_1'}]\, e^{\lambda_i (T_2 - T_1')} = \xi_i^*(T_1')\, e^{\lambda_i (T_2 - T_1')} + \delta\xi_i(0)\, e^{\lambda_i T_2}. \tag{13}$$

Since all the $\delta\xi_i$ are very small, the terms of Eq. (13) which depend on $T_1'$ can be computed approximately. Thus, for example,

$$\xi_1^*(T_1') \approx \xi_1^*(T_1) + \left(\frac{d\xi_1^*}{dt}\right)_{T_1} \delta T_{(1)},$$

but in the interval $0 < t < T_1'$,

$$\frac{d\xi_1^*}{dt} = y = k$$

and

$$\xi_1^*(T_1') = \xi_1^*(T_1) + k\delta T_{(1)} = \xi_1^*(T_s) + k\delta T_{(1)}. \tag{14}$$

Similarly, we get

$$\begin{aligned}\dot{\xi}_i^*(T_1')\,e^{\lambda_i(T_s-T_1')} &\approx \xi_i^*(T_1)\,e^{\lambda_i(T_s-T_1)} + e^{\lambda_i(T_s-T_1)}\left[\left(\frac{d\xi_i^*}{dt}\right)_{T_1} - \lambda_i\xi_i^*(T_1)\right]\delta T_{(1)} = \\ &= \xi_i^*(T_1)\,e^{\lambda_i(T_s-T_1)} + e^{\lambda_i(T_s-T_1)}k\delta T_{(1)} = \xi_i^*(T_s) + e^{\lambda_i(T_s-T_1)}k\delta T_{(1)}.\end{aligned} \tag{15}$$

Substituting (14) and (15) in (13) we obtain

$$\xi_1(T_s) = \xi_1^*(T_s) + k\delta T_{(1)} + \delta\xi_1(0),$$

$$\xi_i(T_s) = \xi_i^*(T_s) + e^{\lambda_i(T_s-T_1)}k\delta T_{(1)} + e^{\lambda_i T_s}\delta\xi_i(0)$$

or

$$\begin{aligned}\delta\xi_1(T_s) - \delta\xi_1(0) &= k\delta T_{(1)}, \\ \delta\xi\ (T_s) - \delta\xi_i(0)\,e^{\lambda_i T_s} &= e^{\lambda_i(T_s-T_1)}k\delta T_{(1)}.\end{aligned} \tag{16}$$

We now consider the process in the interval $T_s < t < 2T_s$. After $t = T_s$ the pulse length equals $T_1'' = g|x(T_s)| = -g(x)(T_s)$, since $x^*(T_s)$ and, consequently, $x(T_s)$ are negative. Letting $T_1'' = T_1 + \delta T_{(2)}$, we get

$$\delta T_{(2)} = -\sum_{i=1}^{n} G_i\delta\xi_i(T_s).$$

If we wish to retain Formula (12), i.e., to set

$$\delta T_{(2)} = \sum_{i=1}^{n} G_i\delta\xi_i(T_s),$$

then we must have a minus sign in the right members of Eq. (16). Moreover, with this, $\gamma = -k$ and the quantity $\underline{k}$ in the right members of Eq. (16) must be replaced by $-k$. As is obvious, Formulas (16) do not change. In general,

$$\begin{aligned}\delta\xi_i(nT_s) - e^{\lambda_i T_s}\delta\xi_i[(n-1)T_s] &= ke^{\lambda_i(T_s-T_1)}\delta T_{(n)}, \\ \delta T_{(n)} &= +\sum_{i=1}^{n} G_i\delta\xi_i[(n-1)T_s].\end{aligned} \tag{17}$$

Equations (17) are difference equations with constant coefficients. Let the solution of (17) have the form

$$\delta\xi_i(nT_s) = M_i\rho^n,\ \delta T_{(n)} = N\rho^n.$$

Substituting this in (17) we obtain

$$M_i(1 - e^{\lambda_i T_s}\rho^{-1}) = kNe^{\lambda_i(T_s-T_1)}, \tag{18}$$

$$N = \sum_{i=1}^{n} G_i M_i\rho^{-1}. \tag{19}$$

From (18) and (19), taking (11) into account, we get

$$1 = gk\left[-\frac{1}{Q(0)(\rho-1)} - \frac{e^{\lambda_i(T_s-T_1)}}{\lambda_i Q'(\lambda_i)}\,\frac{1}{(\rho - e^{\lambda_i T_s})}\right]. \tag{20}$$

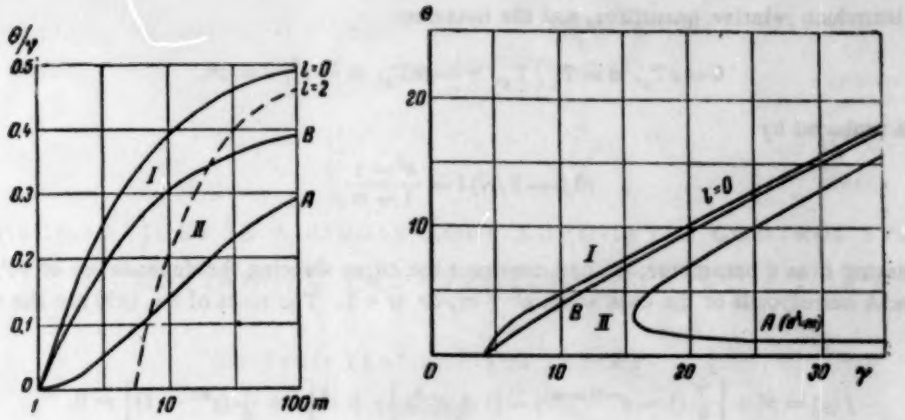


Fig. 4

The stability conditions reduce to the requirement that all the roots $\rho_1$ of (20) satisfy the condition

$$|\rho_1| < 1.$$

We now consider the case when $gx(0) > T_3$. With this, $T_1 = T_3 = \text{const}$. Here,

$$gx(0) = \sum_{i=1}^{n} G_i \xi_i(0) = gx^*(0) + \sum_{i=1}^{n} G_i \delta\xi_i(0) > T_3.$$

Since all the $\delta\xi_1$ are very small, the pulse lengths, as before, equal T, and we may write

$$\xi_1(T_3) = \xi_1^*(T_3) + \delta\xi_1(T_3) = \xi_1^*(T_3) + \delta\xi_1(0),$$

$$\xi_i(T_3) = \xi_i^*(T_3) + \delta\xi_i(T_3) = \xi_i^*(T_3) + \delta\xi_i(0)\, e^{\lambda_i T_3},$$

$$g|x(T_3)| = g\left|gx^*(T_3) + \sum_{i=1}^{n} G_i \delta\xi_i(0)\, e^{\lambda_i T_3}\right| > T_3,$$

i.e., the length of the second pulse, as before, equals $T_3$. In addition to $\delta\xi_1(0)$, $\delta\xi_1(0)e^{\lambda_1 t}$ also decreases and, therefore, after a sufficiently long time, $x(t) = x^*(t) + [1/g)(G_1\delta\xi_1(0)]$. If we assume that the perturbations are random, we may consider that some perturbations increase the deviations and some decrease them. Therefore, after a k-fold action of the perturbations, $\Sigma G_1 \delta\xi_{1k}(0)$ will also be small, from whence we may determine that the oscillation is stable.

## 4. Decomposition of Parameter Space Into Regions

We consider the simplest problem in which

$$Q(s) = s + a.$$

With this, $T_1$ satisfies the equation

$$\left(\frac{1}{2a} - \frac{1}{gk}\right) T_1 = \frac{1}{a^2} \frac{e^{aT_1} - 1}{1 + e^{aT_3}}. \tag{21}$$

Equation (20) is written in the form

$$1 = gk\left[-\frac{1}{a(\rho - 1)} + \frac{e^{-a(T_3 - T_1)}}{a(\rho - e^{-aT_3})}\right]. \tag{22}$$

We now introduce relative quantities, and the notation:

$$\theta = aT_s,\ \alpha = T_1 / T_s,\ \gamma = gkT_s,\ m = e^{\theta},\ l = \alpha\theta,$$

and then (21) is replaced by

$$(0{,}5 - \theta / \gamma)\, l = \frac{e^l - 1}{1 + m}. \tag{23}$$

By considering $l$ as a parameter, we can construct the curve showing the dependence of $\theta/\gamma$ on $\underline{m}$ (Fig. 4). On Fig. 4, curve A corresponds to the case when $e^l = m$, or $\alpha = 1$. The roots of Eq. (22) are the roots of the equation

$$f(\rho) = \rho^2 + \left[\frac{\gamma}{\theta}(1 - e^{-\theta(1-\alpha)}) - (1 + e^{-\theta})\right]\rho + l^{\theta}\left[1 + \frac{\gamma}{\theta}(e^{\alpha\theta} - 1)\right] = 0.$$

The condition for stable oscillations has the form: $f(0) < 1$, $f(\pm 1) > 0$.

By taking (23) into account one can show that $f(\pm 1)$ is always larger than zero and that the limits of stable oscillations are determined from the equality

$$f(0) = e^{-\theta}\left[1 + \frac{\gamma}{\theta}(e^{\alpha\theta} - 1)\right] = 1.$$

If we cancel $\gamma/\theta$ from (23) we obtain

$$lm^2 - 2(e^l - 1)(1 + l)m + [2(e^l - 1)(1 - l) - l] = 0. \tag{24}$$

If we are given $l$, we can compute $\underline{m}$ from (24). We thus obtain curve B (Fig. 4). We can decompose both the ($\gamma/\theta$, m) and the ($\gamma$, $\theta$) planes into two subplanes each. Subplane I corresponds, in each case, to stable oscillations, and subplane II to unstable oscillations. We might mention that if $\underline{g}$ is very small, the system corresponds to Tsypkin's first type of sampled-data system in which the limit of stability is defined by the curve $l = 0$ on Fig. 4.

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# AUTO-OSCILLATIONS IN A SINGLE-LOOP AUTOMATIC CONTROL SYSTEM CONTAINING TWO SYMMETRIC RELAYS*

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By the use of Ya. Z. Tsypkin's method [1], the exact equations are derived for the determination of the symmetric states in a single-loop circuit which contains two symmetric relays. It is shown that, if one assumes that the relays are separated by harmonic filters, then the exact method leads to the same results as the approximate method based on harmonic balance, which is used by E. M. Nadzhafov [2].

Ya. Z. Tsypkin [1], in considering the exact determination of the symmetric auto-oscillations in a single-loop system containing one relay, showed that if one assumes that the linear part of the system is an ideal filter, then the exact methods for determining the periodic regimens (but not the conditions for their stability) lead to the same results as the harmonic balance method.

In the present work, analogous results are obtained for a single-loop system containing two symmetric ideal relays. For this, the exact solution is derived by the method of Ya. Z. Tsypkin [1], but the harmonic balance method for the system with two nonlinearities is used in the form given by E. M. Nadzhafov [2].**

## 1. The Exact Solution

We shall consider the single-loop system (Fig. 1) described by the equations

$$y_1 = f(x_1),\ y_2 = f(x_2),$$
$$x_2 = W_1(p)\,y_1,\quad x_3 = W_2(p)\,y_2,\quad x_1 = -x_3, \tag{1}$$

where $f(x_1)$ and $f(x_2)$ are ideal relay characteristics (Fig. 2) and $W_1(p)$ and $W_2(p)$ are the transfer functions of the linear portions of the system (Fig. 1).

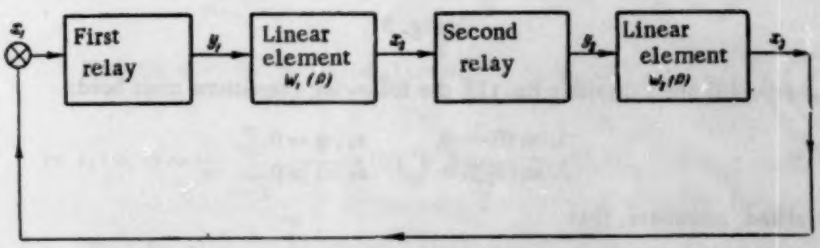


Fig. 1

*This work was carried out under the direction of M. A. Aizerman.

**The exact solution for the problem of symmetric oscillations of the system containing two ideal relays was obtained by Yu. I. Neimark [3], using other methods.

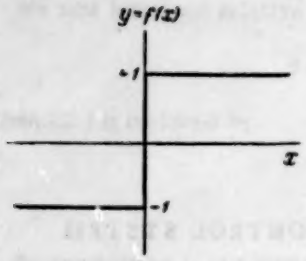

Fig. 2

We are interested in the symmetric oscillations and, following Ya. Z. Tsypkin's method, we assume that the form of the oscillations of the coordinates $y_1$ and $y_2$ is as shown in Fig. 3. Then the functions $y_1(t)$ and $y_2(t)$ can be expressed by Fourier series

$$y_1(t) = \frac{4}{\pi}\sum_{m=1}^{\infty}\frac{1}{2m-1}\sin(2m-1)\,\omega t,$$
$$y_2(t) = \frac{4}{\pi}\sum_{m=1}^{\infty}\frac{1}{2m-1}\sin(2m-1)\,\omega(t-\tau), \tag{2}$$

and the functions $x_2(t)$ and $x_3(t)$ by the series

$$x_2(t) = \frac{4}{\pi}\sum_{m=1}^{\infty}\frac{|W_1[i(2m-1)\omega]|}{2m-1}\sin\{(2m-1)\,\omega t + \arg W_1[i(2m-1)\omega]\},$$
$$x_3(t) = -x_1(t) = \frac{4}{\pi}\sum_{m=1}^{\infty}\frac{|W_2[i(2m-1)\omega]|}{2m-1}\sin\{(2m-1)\,\omega(t-\tau) + \arg W_2[i(2m-1)\omega]\}. \tag{3}$$

Therefore,

$$\frac{dx_2(t)}{dt} = \frac{4\omega}{\pi}\sum_{m=1}^{\infty}|W_1[i(2m-1)\omega]|\cos\{(2m-1)\,\omega t + \arg W_1[i(2m-1)\omega]\},$$
$$\frac{dx_3(t)}{dt} = -\frac{dx_1(t)}{dt} = \frac{4\omega}{\pi}\sum_{m=1}^{\infty}|W_2[i(2m-1)\omega]|\cos\{(2m-1)\,\omega(t-\tau) + \arg W_2[i(2m-1)\omega]\}. \tag{4}$$

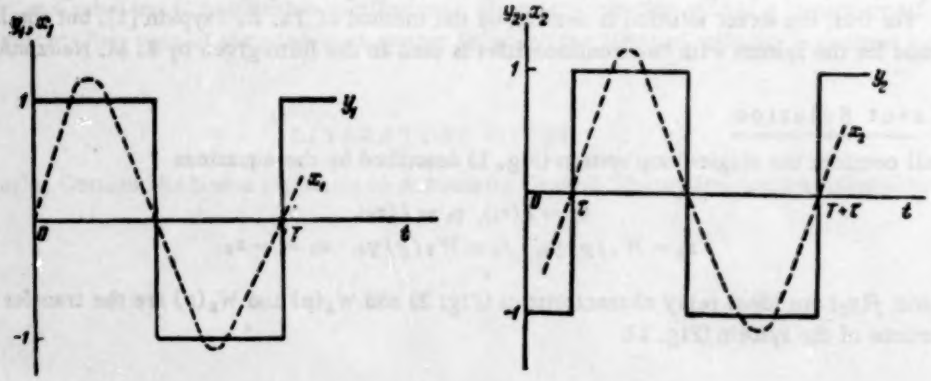

Fig. 3

In order that series (2) and (3) satisfy Eq. (1), the following conditions must hold:

1. $x_1(0) = 0, \qquad x_2(\tau) = 0,$
2. $\dot{x}_1(0) > 0, \qquad \dot{x}_2(\tau) > 0.$

It must be verified, moreover, that

3. $x_1(t) > 0$ for $0 < t < \frac{T}{2}$,

$x_2(t) > 0$ for $\tau < t < \frac{T}{2} + \tau$, $\left(T = \frac{2\pi}{\omega}\right)$.

The first and second conditions lead to the following equalities and inequalities:

$$\frac{4}{\pi}\sum_{m=1}^{\infty}\frac{|W_2[i(2m-1)\omega]|}{2m-1}\sin\{(2m-1)\omega(-\tau)+\arg W_2[i(2m-1)\omega]\}=0, \tag{5}$$

$$\frac{4}{\pi}\sum_{m=1}^{\infty}\frac{|W_1[i(2m-1)\omega]|}{2m-1}\sin\{(2m-1)\omega\tau+\arg W_1[i(2m-1)\omega]\}=0, \tag{6}$$

$$\frac{4\omega}{\pi}\sum_{m=1}^{\infty}|W_2[i(2m-1)\omega]|\cos\{(2m-1)\omega(-\tau)+\arg W_2[i(2m-1)\omega]\}<0, \tag{7}$$

$$\frac{4\omega}{\pi}\sum_{m=1}^{\infty}|W_1[i(2m-1)\omega]|\cos\{(2m-1)\omega\tau+\arg W_1[i(2m-1)\omega]\}>0. \tag{8}$$

The solutions $\omega_0$ and $\tau_0$ of Eqs. (5) and (6), which satisfy inequalities (7) and (8), define the functions $x_1(t)$ and $x_2(t)$, which satisfy conditions 1 and 2. If, with this, conditions 3 hold, then the system under consideration will have a closed trajectory in phase space; if this is a stable limiting cycle, then the sought autooscillation of frequency $\omega_0$ can arise in the system.

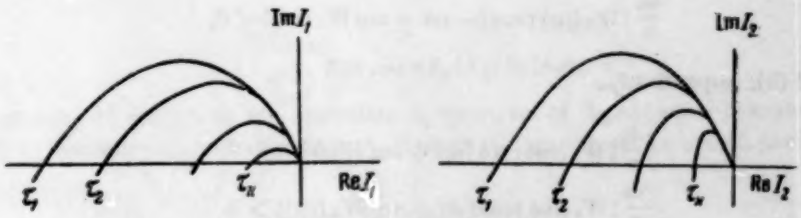


Fig. 4

If we introduce the following "relay system characteristics" we can present the equalities and inequalities in (5)-(8) in a form more convenient for solution:

$$I_1(\omega,\tau)=-\frac{1}{\omega}\dot{x}_1(0)-ix_1(0),\quad I_2(\omega,\tau)=-\frac{1}{\omega}\dot{x}_2(\tau)-ix_2(\tau).$$

By introducing the real and imaginary frequency characteristics $U_j(\omega)$ and $V_j(\omega)$ of the linear portions of the system

$$W_j(i\omega)=U_j(\omega)+iV_j(\omega)\quad(j=1,2), \tag{9}$$

we may write conditions (5)-(8) in the form

$$\operatorname{Im} I_1(\omega,\tau)=\frac{4}{\pi}\sum_{m=1}^{\infty}\frac{1}{2m-1}[-U_2(\omega_m)\sin\omega_m\tau+V_2(\omega_m)\cos\omega_m\tau]=0, \tag{10}$$

$$\operatorname{Re} I_1(\omega,\tau)=\frac{4}{\pi}\sum_{m=1}^{\infty}[U_2(\omega_m)\cos\omega_m\tau+V_2(\omega_m)\sin\omega_m\tau]<0, \tag{11}$$

$$\operatorname{Im} I_2(\omega,\tau)=\frac{4}{\pi}\sum_{m=1}^{\infty}\frac{1}{2m-1}[U_1(\omega_m)\sin\omega_m\tau+V_1(\omega_m)\cos\omega_m\tau]=0, \tag{12}$$

$$\operatorname{Re} I_2(\omega,\tau)=\frac{4}{\pi}\sum_{m=1}^{\infty}[U_1(\omega_m)\cos\omega_m\tau-V_1(\omega_m)\sin\omega_m\tau]<0, \tag{13}$$

where $\omega_m=(2m-1)\omega$.

Being given various value of $\tau$, one can construct the graphs of $I_1(\omega,\tau)$ and $I_2(\omega,\tau)$ (Fig. 4). The points in which the curves $I_1(\omega,\tau)$ intersect the negative real semiaxis correspond to a series of values of $\tau$ and $\omega$; the

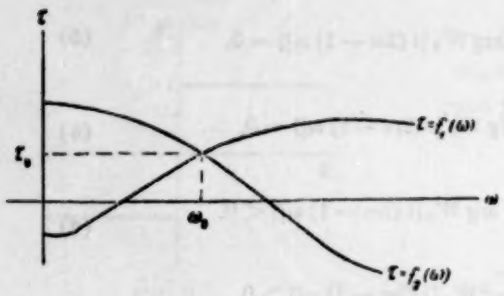


Fig. 5

analogous points for the curves $I_2(\omega,\tau)$ correspond to another $\tau$, $\omega$ series. If we now construct the curves $\tau = f_1(\omega)$ and $\tau = f_2(\omega)$ on the $\tau$, $\omega$ plane, we obtain the desired $\tau_0$, $\omega_0$ as the coordinates of their point of intersection (Fig. 5).

## 2. The Approximate Solution, and its Comparison with the Exact One

We assume here that both linear portions of the system are ideal filters which pass only the first harmonics of the periodic functions $y_1(t)$ and $y_2(t)$.

It is then necessary, in series (3), to consider only the first terms of the sums in the determination of $x_2$ and $x_3$. Conditions (5) and (7) take the form:

$$\frac{4}{\pi}|W_2(i\omega)|\sin[-\omega\tau+\arg W_2(i\omega)]=0, \tag{14}$$

$$\frac{4\omega}{\pi}|W_2(j\omega)|\cos[-\omega\tau+\arg W_2(i\omega)]<0, \tag{15}$$

and Conditions (6) and (8), respectively,

$$\frac{4}{\pi}|W_1(i\omega)|\sin[\omega\tau+\arg W_1(i\omega)]=0, \tag{16}$$

$$\frac{4\omega}{\pi}|W_1(i\omega)|\cos[\omega\tau+\arg W_1(i\omega)]>0. \tag{17}$$

It follows from (14) and (15) that

$$-\omega\tau+\arg W_2(i\omega)=\pi. \tag{18}$$

It follows from (16) and (17) that

$$\omega\tau+\arg W_1(i\omega)=0. \tag{19}$$

By summing (18) and (19) we find that

$$\arg W_1(i\omega)+\arg W_2(i\omega)=\pi. \tag{20}$$

Equation (20) can be used for determining the frequency $\omega_0$ by auto-oscillation. However, we find it more convenient to replace it by the equivalent system of two equations:

$$\sin[\arg W_1(i\omega)+\arg W_2(i\omega)]=0,$$
$$\cos[\arg W_1(i\omega)+\arg W_2(i\omega)]<0$$

or

$$U_1(\omega)V_2(\omega)+U_2(\omega)V_1(\omega)=0,$$
$$U_1(\omega)U_2(\omega)-V_1(\omega)V_2(\omega)<0. \tag{21}$$

We will now seek the frequency of auto-oscillation by the harmonic balance method. As is known [2], for the determination of $\omega_0$, in this case, we have the equation

$$\mathrm{Im}I(i\omega)=0 \tag{22}$$

and the condition

$$\mathrm{Re}I(i\omega)<0. \tag{22'}$$

In our case, the reciprocal of the frequency characteristic is defined by the expression

$$I(i\omega)=\frac{1}{W(i\omega)},$$

where

$$W(i\omega) = W_1(i\omega)\,W_2(i\omega) = U(\omega) + iV(\omega),$$

where we have taken $U(\omega) = U_1(\omega)U_2(\omega) - V_1(\omega)V_2(\omega)$ and $V(\omega) = U_1(\omega)V_2(\omega) + U_2(\omega)V_1(\omega)$.

Consequently,

$$I(i\omega) = \frac{U(\omega)}{U^2+V^2} - i\,\frac{V(\omega)}{U^2+V^2}.$$

Then, (22) and (22') take the form:

$$\begin{aligned} V(\omega) &= U_1(\omega)\,V_2(\omega) + U_2(\omega)\,V_1(\omega) = 0, \\ U(\omega) &= U_1(\omega)\,U_2(\omega) - V_1(\omega)\,V_2(\omega) < 0. \end{aligned} \tag{23}$$

We now determine the amplitudes $A_1$ and $A_2$ of the periodic functions $x_1(t)$ and $x_2(t)$ by the method of harmonic balance. We find $A_1$ from the conditions of amplitude balance

$$I(i\omega) = -R(A, \omega),$$

where

$$R(A,\omega) = R_1(A_1)\,R_2(A_2).$$

To determine $A_1$ we set $\omega = \omega_0$ and eliminate $A_2$ by means of the equation describing the passage of the sinusoidal signal of frequency $\omega_0$ and amplitude $A_1$ through the portion of the circuit made of of "first relay-first filter":

$$A_2 = |W_1(i\omega_0)|\,R_1(A_1)\,A_1. \tag{24}$$

In our case, the equivalent transfer function of the first relay elements is $R_1(A_1) = 4/\pi A_1$ and Eq. (24) can be used for the determination of $A_2$

$$A_2 = |W_1(i\omega_0)|\,\frac{4}{\pi}. \tag{25}$$

We determine $A_1$ analogously:

$$A_1 = |W_2(i\omega_0)|\,\frac{4}{\pi}. \tag{26}$$

A comparison of (23) with (21) convinces us that the equations for determining the auto-oscillation frequency obtained by the exact method (if the additional assumption is made that each of the linear portions of the system is an ideal filter) coincide with the equations obtained by the harmonic balance method.

A comparison of the amplitudes $A_1$ and $A_2$, determined in accordance with (25) and (26), with the amplitudes of the first harmonics of $x_1$ and $x_2$ which can be obtained from (3) also convinces us that, in the case we have been considering, they also coincide.

From what has been presented we may also draw the converse inference: if at least one of the linear portions passes at least one of the higher harmonics produced by the relays, then the exact and approximate equations will not coincide.

We note in conclusion that the conclusions drawn may easily be extended to single-loop systems containing an arbitrary number of symmetric relays which are separated by harmonic filters.

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# CARBORUNDUM THERMOCOMPENSATORS

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(Leningrad)

High-ohmic and low-ohmic thermocompensating electrical impedances have been made of carborundum which allow heating up to 300°C and power dissipation up to 24 watts.

There have been developed, in the Leningrad Institute for Refractory Materials, low-ohmic (from 5 to 20 ohms) and high-ohmic (from 400 to 4000 ohms) thermocompensating impedances which, in the temperature range from −60 to +300°C, possess negative temperature coefficients equal, for 20°C, to 0.4 + 0.6% per °C. These impedances can dissipate power of up to 25 watts.

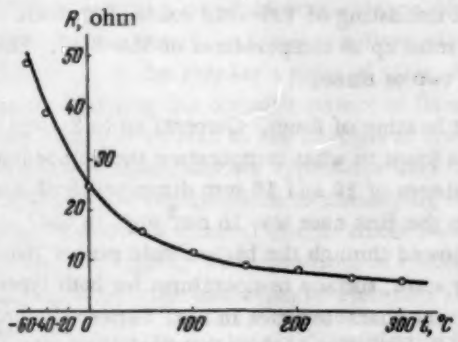


Fig. 1. Impedance variation as a function of ambient temperature.

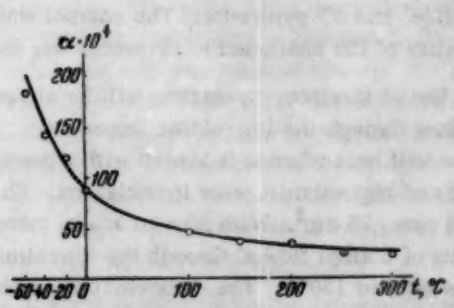


Fig. 2. Temperature coefficient variation as a function of ambient temperature.

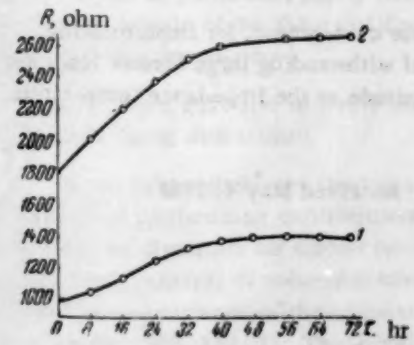


Fig. 3. Impedance variation as a function of work-piece exposure time at 300°. Curve 1 is for an initial impedance of 1000 ohms, and curve 2 is for an initial impedance of 1800 ohms.

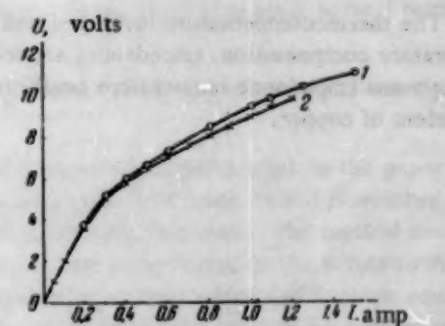


Fig. 4. Volt-ampere characteristics of low-ohmic work-pieces. Curve 1 is for a piece with 10 ohm impedance, and curve 2 is for a 25-ohm piece.

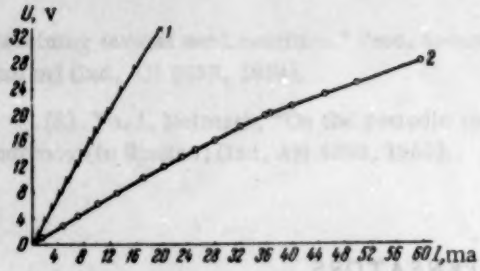


Fig. 5. Volt-ampere characteristics of high-ohmic work pieces. Curve 1 is for a 500-ohm piece, and curve 2 is for a 4000-ohm piece.

These thermocompensators are prepared from black carborundum, elemental silicon, titanium dioxide and kaolin. The carborundum, titanium dioxide and silicon are used in invariable proportions – 40:30:10. The kaolin content varies from 10% to 40%. The moistened mixture, after stirring, is loaded into a hydraulic broaching press, from which it is extruded through a spout 15 to 17 mm in diameter. The extruded material is cut into lengths of 65 to 70 mm and, after being dried, is baked in a coke charge in furnaces with oil heating with a temperature of 1200 to 1300°C for a period of 3 hours. Contact strata are deposited on the ends of the roasted pieces by spraying on aluminum leads in a Schoope apparatus.

Figures 1 and 2 show the dependence of the electrical impedance R and the temperature coefficient of impedance $\alpha$ on the ambient temperature. The measurements were made in the temperature range of −60°C to +300°C. In this interval the variation of impedance with temperature was exponential. The thermocompensators of carborundum changed their impedance in the first hour of operation. It is necessary to maintain the work-pieces at a temperature of 300° for some 50 to 60 hours in order for them to attain a state at which variation of the electrical characteristics of the compensators ceases. Fig. 3 shows the curves of impedance variations as functions of time of exposure to 300° in a furnace.

The behavior of the thermocompensators was studied in a medium with 98% relative humidity. Exposure to such a medium for three to five days resulted in an increase of impedance by 10 to 30%. To protect the work-pieces from the action of humidity, they were sheathed in an enamel consisting of 70% lead oxide, 25% boric anhydride, and 5% pyrolusite. The enamel was sufficiently heat-resistant up to temperatures of 300-350°. The reliability of the enameled work-pieces was increased by a factor of two or three.

Use of thermocompensators will be accompanied by significant heating of them. Currents up to 2 amps may flow through the low-ohmic impedances. It is very important to know to what temperature the compensator surface will heat when it is loaded with a power of 25 watts. Work-pieces of 10 and 16 mm diameter, both low-ohmic and high-ohmic, were investigated. The dissipating surface in the first case was 16 $cm^2$ and, in the second case, 25 $cm^2$. With 25 watt loads, currents up to 0.25 amps flowed through the high-ohmic pieces and currents of 2 amps flowed through the low-ohmic ones. In the steady state, surface temperatures for both types did not exceed 150°C. The compensators retained their stable electrical characteristics in use. Periodic or prolonged loading with power up to 25 watts did not lead to any noticeable variation in the electrical characteristics of the previously aged work-pieces.

Figure 4 gives the volt-ampere characteristics for low-ohmic work-pieces, while Fig. 5 provides them for high-ohmic pieces.

The thermocompensators developed can be used successfully in those cases where, for implementing temperature compensation, impedances are required which are capable of withstanding large current loads and which possess impedance temperature coefficients equal in absolute magnitude to the impedance temperature coefficient of copper.

Received May 4, 1958

# CHRONICLE

## SEMINAR ON THE TECHNICAL APPLICATIONS OF MATHEMATICAL LOGIC (1957-1958)*

During 1957 and 1958 the seminar on the technical applications of mathematical logic continued its meetings, devoting them almost entirely to the discussion of new original work by the seminar participants.

During this time there were 16 sessions, of which 14 were devoted to original work and one to an instructive presentation of the translations of Soviet literature on the seminar subject matter in the USA. The work of the seminar participants was principally devoted to methods of analysis and synthesis of contact, and contactless, relay circuits. Foreign scientists took part in the work of the seminar.

On September 20, 1957, in his paper, "On the fundamental concepts of relay-contact circuit theory," devoted to general methodological questions in relay-contact circuit theory, G. N. Povarov expressed the opinion that in existing relay-contact circuit theory and, in particular, in the terminology recommended in 1953 by the KTT AN SSSR, a two-fold ambivalence of concepts can be observed: the structural point of view is mingled with the functional, and concepts from electrical engineering are mingled with information theoretical ones. According to the speaker's point of view, the functioning of a relay-contact circuit can be reduced to the processes of changing the circuit's state – to its own type of absolutely inflexible nonhomogeneous Markoff chains. The concepts related to the changes in state of the circuit as a whole have, not a structural, but a purely functional character, and are applicable also to contactless relay circuits and to other switching systems. Such, for example, are the questions in classifying sequential relay-contact circuits or questions of determining the minimum number of internal circuit states necessary for storing, in sequential circuits, the necessary information as to previous states. Even questions of coding the circuit's state by combinations of states of its elements, and questions of interconnection of elements are amenable to treatment with structural concepts.

Further, in the speaker's opinion, relay-contact circuits are essentially devices for the processing (transformation) of information, for which engineering questions are only means, not ends. While admitting the possibility of treating relay-contact circuit theory as a branch of nonlinear electrical engineering, the speaker still asserted that a more general relay system theory can only be a portion of the theory of discrete transformers of information. In particular, purely functional questions in the theory of relay-contact circuits must be treated just in the domain of the theory of discrete information transformers. The speaker also made several remarks anent existing terminology.

The paper gave rise to lively discussion. Present for the paper was the American scientist Koni, then in the Soviet Union as a tourist.

V. N. Grebenshchimov (post-graduate physics student at the Moscow State University), in the paper "Methods of synthesizing multiterminal contact circuits of the general type," (October 25 and November 1, 1957), presented an algorithm for circuit synthesis using a method he had previously presented. The method consists of a successive removal of redundant contacts from the sought multiterminal network and in the determination of the contact admittance of the remaining portions of the circuit by solving certain systems of Boolean equations (DAN SSSR, 1958, 119, 2). The algorithm consists in writing, in some general form and using a special symbolism, all the solutions to each system of equations and in then applying three types of selection rules. Use of the

*Proceedings of the seminar prior to 1957 are reported in Automation and Remote Control (USSR) 18, 10 (1957).

first type of rule eliminates those superfluous solutions which, by different paths, would lead to identical circuits. The application of the second type of rule leads to the elimination of those solutions which would lead to circuits which could be transformed into each other by renumbering the variables in the functions or by repositioning the contacts in the circuit without disrupting the contact admittance realized by the circuit. The use of the third type of rule leads to the elimination of all the remaining solutions but one. Changing the third type of selection rule allows one to obtain circuits which satisfy various requirements.

Methods of analyzing multiterminal contact circuits were treated in the papers of V. N. Grebenshchikov, "A method of analyzing multiterminal contact circuits, and its mechanization," (Feb. 6, 1958), "A method of analyzing multiterminal contact circuits with rectifying elements" (March 6, 1958), and "A method of analyzing multiterminal circuits consisting of contact multipoles" (March 20, 1958).

The first of these papers presented an analytical method, suggested by the author, for multiterminal contact circuits, the method consisting of successive determinations of the contact admittances of chains of subcircuits of the given circuit, each of which subcircuits is formed from the previous chain either by the operation of joining one contact to two terminals of the circuit or by the operation of introducing one free terminal. To determine the contact admittance of each subcircuit, one uses a system of Boolean equations which relates the admittance of the new subcircuit with the contact admittance of the previous chain. Also presented was a method of mechanizing the foregoing method, which consisted of coding the Boolean functions in the binary number system (one number to each term of a Boolean function) and a reduction of the algebraic operations for transforming contact admittances to a set of machine operations on numbers.

In the second paper, the method just presented was extended to circuits with rectifying elements by the use of functions of direct and reverse admittance for each terminal pair and by adding the operation of joining a rectifier to two terminals of the circuit.

In the third paper this method was extended to circuits of contact multipoles with given contact admittances between the terminals of these multipoles. For this there was introduced the operation of connecting to two terminals of the circuit an element which posses a constant contact admittance of 1 in both directions.

M. L. Tsetlin (post-graduate student in physics at the Moscow State University), in the paper "A matrix method for synthesizing relay-contact and electronic circuits," (December 19 and 26, 1957), presented a method suggested by himself for the synthesis of primitive and nonprimitive relay-contact and electronic relay circuits [DAN SSSR 117, 6 (1957) and 118, 3 (1958); Elektrosvyaz' 4 (1958)].

The author calls a relay circuit primitive if the set $X_t$ of quantities $x_t^1, \ldots, x_t^{n+s}$ which determine the states of the circuit's input busses completely determines the states of the circuit's output busses, i.e., if the following relationships hold:

$$f_t^i = f^i(x_t^1, \ldots, x_t^{n+s}) \qquad (i = 1, 2, \ldots, p+s).$$

The author calls a circuit nonprimitive if it is obtained from a primitive circuit by the introduction of a feedback loop with unit time delay:

$$x_{t+1}^{n+i} = f_t^{p+i} \qquad (i = 1, 2, \ldots, s),$$

i.e., by the identification of the states of the input busses $x_{t+1}^{n+i}$ at time t + 1 with the states of the output busses $f_t^{p+i}$ at time t.

By use of a matrix method certain general properties of such circuits were established, and formulas were deduced which permit the structure of the circuit to be synthesized to be determined directly from the matrices.

In the paper there was given a detailed presentation of a method for synthesizing nonprimitive circuits constructed of electromagnetic relays and triggers. The method is also easily generalized to circuits constructed from other elements.

In the paper of M. L. Tsetlin and L. M. Shekhtman (NIISChETMASh Gosplans SSSR), "Ferrite-transistor circuits and methods of synthesizing them" (April 3 and 10, 1958), circuits were described which contained ferrite cores and transistors, principles were presented for the design of universal elementary cells which realize the function $x\bar{y}$, and a methodology was presented for the synthesis of circuits constructed from these cells.

The methodology for circuit synthesis consists of the introduction of a special symbolism for the elementary cells and in the rules for the superposition of the elementary functions realized by the cells. It was shown that an arbitrary function can be realized in not more than two cycles (stages) of a device, and the necessary formulas were obtained. A technique for minimizing transformations was described. By the use of special formulas a method was presented for constructing nonprimitive circuits of ferrite-transistor elements directly from the matrices of these circuits. It was noted that the synthetic method presented by the authors was used in the design of digital computers.

In the paper of V. I. Shestakov, "A card method for synthesizing sequential relay systems," (September 27, 1957), the author gave his method for implementing the vector algebraic method of synthesizing two-position relay systems directly from special punched cards [Automation and Remote Control (USSR) 19, 6 (1958)].*

The vector-algebraic method of synthesizing two-position relay systems, presented in previously published works of the author [Automation and Remote Control 15, 4 (1954); Doklady AN SSSR 98, 4 (1954); and 99, 6 (1954)] always permits the Boolean functions describing the conditions of operation of each relay of the system to be synthesized to be obtained from given sequences of operation of all the relays of this system, plus given sequences of external stimuli acting on this system. However, with a large number of relays in the system to be synthesized and a large number of independent stimuli acting on this system, the computation of the Boolean functions describing the operation of the system's relays requires a large expenditure of work and time. This gives rise to the requirement for alleviating and accelerating this computation by mechanizing at least part of it. The simplest method of effecting this mechanization is the punched-card method presented.

V. I. Shestakov, in his papers, "Simulating the operations of D. A. Bochvar's calculus by three-position switches," (November 22, 1957), and "On one arithmetic interpretation of D. A. Bochvar's three-valued calculus," (November 26, 1957), considered questions of simulating the operations of the three-valued propositional calculus by means of three-position relay-switch circuits.

In the first of these papers the three-valued calculus of D. A. Bochvar was extended to a functionally complete propositional calculus and a method of simulating the operations of this extended calculus by means of three-position switches was considered. In this method, as the simulating parameter and the coefficient of voltage transmission, $k = E_2/E_1$, was chosen, where $E_1$ and $E_2$ are the voltages at the switch's input and output, respectively. The correspondence between the values of the three-valued proposition P and the coefficient $\underline{k}$ was established by the equation k = sign P, where the function sign P, "value of the proposition P," introduced by the author, is defined by the following conditions;

$$\text{sign } P = \begin{cases} 1 \text{ if P is true,} \\ 0 \text{ if P is vacuous} \\ -1 \text{ if P is false} \end{cases}$$

This method of simulating was treated in detail in the paper, "Simulating the operations of propositional calculus by means of relay-contact circuits," which was included in the collection of papers on logic published by the Philosophy Institute of the AN SSSR.

The second paper of V. I. Shestakov was devoted to the author's new arithmetic interpretation of the functionally complete propositional calculus. The previous interpretation was based on the equation given above, k = sign P. The new interpretation is based on the equation k = sign I holding between the current I flowing through the relay winding switched to the output of the three-position switch, and the coefficient $\underline{k}$. By eliminating the coefficient $\underline{k}$ from these two equations, we obtain the equation sign P = sign I, which lies at the base of the new arithmetic interpretation of the three-valued propositional calculus. This equation permits each operation of the functionally complete three-valued propositional calculus to be represented by a corresponding arithmetic operation. Each of these latter operations is realized by some relay-switch circuit.

In the paper of Yu. A. Shreider (Moscow), "On the analysis and synthesis of certain electronic circuits," (December 12, 1957), there was described the work carried out jointly by the author and B. I. Rameev. In this work a somewhat modified mathematical apparatus of the propositional calculus was used in the analysis and synthesis of networks constructed of diodes. Rules were formulated for finding the total number of diodes contained in a given circuit from the structural formulas of the circuit.

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*See C. B. translation.

Taking part in the seminar session of October 12, 1957 were Rumanian scientists led by Academician Gr. K. Moisil. They presented two papers on work on relay circuit theory in the Rumanian National Republic.

In the paper of P.Constantinescu, "On circuits with the structural formula $x_1 \oplus x_2 \oplus \ldots \oplus x_n$," there were presented results which were contained in the paper previously published by the author, "Asupra reducerii numarului de contacte prin introducerea circuitelor in punte," (Analele Universitatii "C. I. Parhon," Bucuresti, No. 11, 1956, Ser. Stiin. Nat).

In the paper of G. Ioanin, "On the synthesis of circuits with relays and stepping switches," there were given methods for synthesizing circuits constructed of stepping switches and relays, these methods having been developed by the author in cooperation with Gr. K. Moisil. As noted by the speaker, he tended to construct algorithmic methods which would be amenable to subsequent mechanization.

The paper of M. V. Surgucheva (Physical Institute, AN LatvSSR), "A survey of digital computers from the point of view of the operations executed by them," (May 15, 1958), contained a general classification of the types of operations executed in electronic digital computers, and estimates of these operations as applied to the practical construction and use of one of these machines.

At the same seminar session (May 15, 1958), G. N. Povarov reported on the translations in the USA of the works of Soviet authors on relay circuit theory. The speaker gave a list of such translations made within the last few years by the translating firm of Morris D. Friedman, Inc. (Needham Heights, Mass., USA).

V. N. Grebenshchikov

## CONFERENCE ON THE THEORY AND APPLICATION OF DISCRETE AUTOMATIC SYSTEMS

From September 22 to 26, 1958 in the Moscow Dome, there was held a conference on the theory and application of discrete automatic systems, organized by the National Committee of the Soviet Union on Automatic Control in conjunction with the Institute for Automation and Remote Control of the AN SSSR. More than 600 scientific workers and engineers from 265 organizations in 24 cities took part in the conference.

The plenary session of the conference was opened by V. A. Trapeznikov, chairman of the National Committee of the Soviet Union on Automatic Control, director of the Institute for Automation and Remote Control of the AN SSSR, and corresponding member of the AN SSSR.

In his introductory remarks, V. A. Trapeznikov stated that the basic task of the conference was a broad discussion of the state of the theory and practical application of discrete automatic systems, and the prospects for their further development.

One of the principal directions in the development of technological processes in our country is the broad development of automata. Automata cannot be successfully developed without being based on scientific achievements.

Recently, the science of automatic control has been concentrated in a new direction, related to the development of the theory and practice of discrete automatic systems. The use of such systems allows flexible control and adjustment of technological processes to be carried out, and allows great economies to be effected in many cases. V. A. Trapeznikov further stated that the present conference should be considered as a preliminary to the International Congress on Automatic Regulation and Control to be held in Moscow in 1960.

The plenary session continued with a paper by Ya. Z. Tsypkin (Moscow) on the subject, "Discrete automatic systems and prospects for their development." He noted that contemporary automation is characterized by complex problems, arising not only from the complexity of the objects of regulation, but from the increased requirements on intensification, economy, and effectiveness of the regulated processes.

The solution of these problems led to the creation of a new class of automatic systems, possessing better qualities than the ordinary automatic control systems. In this class fall the so-called self-adjusting systems.

These systems carry out analyses of the previous state of the process, observe or simulate the present state,

and a prediction of the future state. Execution of these operations requires a large amount of computation, both arithmetical and logical. The advent of digital computers had an essential effect on the development of self-adjusting systems. Instead of the ordinary setting of the given and desired values of the regulated quantities, such systems carry out searches for optimal states (minimum control time, expense, power, maximum productivity, efficiency, etc.).

Ya. Z. Tsypkin dealt in detail with the classification of discrete automatic systems and their characteristic idiosyncrasies. Using as the basis for his classificatory scheme the method of transforming and transmitting information (the method of quantizing physical quantities), the speaker distinguished three basic classes of discrete automatic systems: relay, pulsed, and pulse-relay or digital automatic systems. In relay systems, the physical quantities are characteristically quantized by level, while the quantization is by time in pulsed (sampled-data) systems. In systems using digital computing devices, quantification by level and by time occurs simultaneously.

Of particular interest in Ya. Z. Tsypkin's paper was the theoretical discussion of the principles of quantification and the effects engendered by quantification, such as, for example, the observation of high-frequency periodic signals by means of narrow-band apparatus, stabilization, and improvement of dynamic properties of systems with lags, etc. Considered also were the theoretical bases of reproducing a continuous signal from its values at discrete moments of time, as well as the limitations inherent in time quantification. Quantification by level is a nonlinear process. However, if, instead of signals, one considers their distribution densities, then many of the results applicable to the operation of time quantification become applicable to this case as well.

After a brief survey of the results obtained in discrete system theory [methods of analyzing and synthesizing relay and pulsed (sampled-data) systems, approximate methods for investigating periodic regimens in extremal systems], the speaker formulated a number of problems whose solutions are important for the further development of the theory. Among these problems are: the analysis of self-adjusting systems described by a set of difference or differential equations plus equations describing logical operations; the analysis and comparison of various methods of searching; investigation of methods for coping with noise in self-adjusting systems. Solution of these problems will require the use of results from information theory, the theory of games and dynamic programming theory, decisive for the various optimization problems.

The further work of the conference was carried out in the following sections:

1. Section on the theory of sampled-data and digital control systems, headed by G. S. Pospelov.
2. Section on self-adjusting (extremal) systems, directed by A. A. Fel'dbaum.
3. Section on optimal system theory, headed by A. Ya. Lerner.
4. Section on sampled-data and digital elements and devices, headed by V. A. Kotel'nikov.
5. Section on finite automata theory, headed by M. A. Gavrilov.

The section on the theory of sampled-data and digital control systems contained 11 papers. The speakers covered a broad range of questions.

The paper of G. P. Tartakovskii (Moscow) contained a generalization of the basic theory of sampled-data systems with constant parameters to systems with variable parameters. For the various cases of parameter variation in the linear and pulsed portions, the author introduced the concept of the transmission function depending on time, by means of which the reaction of the system to arbitrary regular input stimuli is found. Also generalized were the stability criteria of systems with constant parameters to systems with variable parameters. Using a pulsed radio range finder as an example, the author showed the methodology of investigating stationary and nonstationary discrete processes.

The paper of V. P. Perov (Leningrad) was devoted to the statistical synthesis of sampled-data systems with variable pulse repetition periods, the variation occurring either in accordance with a previously given rule, or randomly. The speaker considered the possibility of constructing an optimal system where the criterion would be minimum dispersion of the system error. A generalization of the method of synthesizing optimal systems to the case where the variation of pulse repetition period had an arbitrary character allowed the author to draw certain conclusions as to the possibility of constructing self-adjusting optimal systems.

In the paper of Fan' Chun-Wui (Moscow), a method was presented for the analysis of sampled-data systems in which the pulse sequence is set up by a pulse element with complicated modulation: by frequency and by amplitude with weighted coefficients. Based on this method were the considerations of applications of complicated modulation for solving problems of interpolation, for implementing systems which are optimally fast, for increasing noise imperviousness, the quality of signal reproduction, and the degree of system stability.

As one particular case of a sampled-data system with complicated modulation there was analyzed a system containing several pulse elements with different repetition periods.

In his paper, F. M. Kilin (Leningrad) considered exact and approximate methods for analyzing and synthesizing sampled-data systems with variable parameters, in which the variation is stepwise. The author adduced results of an investigation of the statistical dynamics of such sampled-data systems with gating of random signals by a system of coincidence amplifiers, and he also presented criteria for optimal systems based on the conditions of noise passage for the case when the noise is produced in the intervals between control pulses.

In his second paper, Ya. Z. Tsypkin gave a general formulation of the problem of synthesizing the structures of pulsed devices designed to compensate for the effect of object delays on the dynamic characteristics of the servo system. He obtained the conditions for the possibility of complete and of partial lag compensation, and considered the example of synthesizing a compensated optimal system.

A. A. Krasovskii (Moscow) devoted his paper to the synthesis of self-adjusting systems with digital correcting devices. The author showed how to choose the transfer function of the discrete correcting device of a linear control system which would guarantee optimal processes in the closed system for given characteristics of the object and of the executive portion of the regulator. He then presented a method for adding subsidiary correcting links to the self-adjusting system which would guarantee the automatic adjustment of the discrete correcting device when there were changes in the characteristics of the object or the executive organ. The method of synthesis presupposes that the probability characteristics of the noise are such that the correlation time is sufficiently small. With this, the optimal distribution of the weights of the closed-system coefficients is a linear one.

In his paper, E. A. Krogius (Leningrad) gave a frequency method for designing linear sampled-data control systems with random noise. The author obtained simple relationships between the parameters of the correcting device and those of the basic system parts. The method is based on the use of logarithmic frequency characteristics. In the paper were given conditions under which a sampled-data control system can be considered as a continuous one.

In his paper, E. M. Esipovich (Moscow) introduced the concept of the so-called conditional transfer function which, in contradistinction to the ordinary transfer function, depends on the form of the input stimulus. The author showed that the properties of the conditional transfer functions are such that the theory of sampled-data control can be extended to cover them.

Certain features peculiar to conditional transfer functions were illustrated in the example of a discrete system with a linear extrapolating device operating under noisy conditions.

I. V. Pyshkin (Moscow) devoted his paper to the analysis of auto-oscillations in systems with pulse-width modulation. In the paper were given the necessary conditions for the existence of symmetric auto-oscillations, expressed in terms of the frequency characteristics of the continuous portions of the system, and a method of investing the stability of these auto-oscillations was given. For the simplest forms of auto-oscillations, the speaker gave nomograms which permit one to determine, directly from the form of the frequency characteristics of the linear portion, whether the conditions for existence of auto-oscillations hold.

I. I. Perel'man (Moscow) gave, in his paper, the design principles for automatic control systems called, by the author, systems with self-established programs. Such systems guarantee the required responses to the signal components whose durations are either greater, or smaller, than the net object lag. Systems with self-established programs are designed to work with stimuli which repeat in time (are cyclical). As an example, the author considered a system for stabilizing the thickness of hot-rolled steel sheet in a continuous rolling mill. The paper contained an investigation of the stability of a system with a self-established program, and provided data on the choice of the optimal coefficients when the input to the system contained a cyclical stimulus accompanied by random noise.

In the paper of B. A. Sigov (Kiev) there was provided an investigation of position, velocity, and integral systems of digital program control. A comparative investigation of the different structures was given, using as an example the control system for copying machines. The author explained a new structural scheme for a system of digital program control (DPC) which is called "integral." In an integral DPC the integral requiring translating is written, but the translating itself is done automatically. This leads to a definite shortening of program length.

Thirteen papers were presented in the section on self-adjusting (extremal)systems, these papers being devoted to self-adjusting system theory, design principles for extremal systems, and also to descriptions of apparatus tested in the laboratory or in prototype use.

The paper of Yu. V. Dolgolenko (Leningrad) was devoted to the derivation of precise relationships between the parameters of relay extremal systems and the frequency and amplitude of auto-oscillations. The author compared the exact method with the harmonic balance method, previously used for the same purpose by I. S. Morosanov.

As a result of this comparison, Yu. V. Dolgolenko drew the conclusion that it is possible to employ the harmonic balance method in synthesizing systems of the class considered, since the errors introduced by this method for high frequencies are small.

In his paper, I. S. Morosanov (Moscow) considered an approximate frequency method for calculating the steady-state periodic regimens in extremal control systems with independent scans. Expressions were given for the equivalent complex gain for various relay and relay-pulse extremal systems. The speaker showed that the necessary conditions for the existence of auto-oscillations may be determined graphically from the equivalent frequency characteristics of the object.

Yu. I. Ostrovskii (Moscow) devoted his paper to a method of designing extremal systems when noise is taken into account. The method was illustrated by the example of the design of a pneumatic extremum-regulator. The essential point of the method is the following. Initially, the system is considered without noise at its output and with a given sinusoidal stimulus at the input. In such a system, the harmonic balance method allows one to find the errors accompanying this input excitation. Then, separately, one finds the probability of an incorrect step, or the frequency of incorrect steps, at the object output when noise is present. The system parameters are then so chosen that, for a given error, the attendant frequency of incorrect steps will be a minimum. The author provided experimental verification of the results of his theoretical investigation.

The paper of V. V. Kazakevich (Moscow) considered questions of stability and quality of extremal control systems, and also the principles of an extremal controller with a commutator implementing a forced reversal, and methods of introducing dummy extremum indicators.

V. V. Kazakevich questioned the feasibility of extremal controllers which are developed without external disturbances being taken into consideration. If the speed of variation of the external excitations is commensurate with the speed of processing of the stimuli to be regulated, the system may turn out to be unstable and, in order to avoid incorrect system operation and its passage from an extremum to a limiting position, it would be necessary to carry out periodically a forced reversal. The speaker suggested that, in those cases where the characteristic either has a bounded limb or no extremal indicator expressed at all, the characteristic be transformed, by the introduction of additional connections, so that the extremal indicator would become perceptible. Among the methods of improving the quality of pulsed-data systems there is the use of a variable processing step, by means of "steregushchikh" contacts, which do not permit extrema to occur abruptly.

The paper of V. M. Kuntsevich (Kiev) contained a frequency method for synthesizing extremal systems of the stepping type, the method being convenient for engineering computations. Considered in the paper was the block schematic of a system, developed at the Institute for Electrical Technology of the AN SSSR, and data were given on the introduction of this system into industrial use for the maintenance at the maximum level of the efficiency coefficient of the steam boiler of a generating station. Work on the installed regulator is now at the stage of experimental testing. The first trials of the regulator working with an object, carried out jointly by the author and Yu. M. Bulavitskii, gave encouraging results.

L. N. Fitsner (Moscow), in his paper, described two forms of automatic optimizers: the simplest relay type and the proportional action type with small additional perturbations introduced into the system during the scanning

process. The author showed that, in an inertial system operating with a relay optimizer, three fundamental forms of motion are possible, and he found the relationships between the object and optimizer parameters for which the scan process is stable and the amplitude of hunting is minimum.

The paper of E. K. Krug (Moscow) treated of extremal systems for the regulation of objects with respect to two and more parameters. The method for finding extrema is based on the determination of partial derivatives at discrete moments of time, followed by an incrementing of the input quantities proportionally to these derivatives. The paper contained recommendations for the choice of amplitude, phase and frequency of the harmonic components of the input quantities, coefficients of proportionality between the increments of the input quantities and the values of the derivatives obtained as functions of the static characteristics and inertia of the controlled object, and also contained a short description of a regulator which provides an extremum in accordance with the method cited.

In the presentation of his second paper, V. V. Kazakevich gave the results of an investigation of the nonlinear processes in extremal regulators. The Galerkin method underlay the investigation into the steady-state and transient processes in the nonlinear extremal control system. In getting the form of the periodic solutions (including those different from the harmonics) for given boundary conditions, the author obtained a complete description of the processes.

The paper of A. Fel'dbaum (Moscow) was devoted to the synthesis of optimal, or near-optimal, extremal control systems, in the sense of minimum duration of the transient response and maximum accuracy of the extremum determined, taking noise into account. The average scan time of such a system is composed of a number of discrete steps necessary for reaching an extremum from given initial conditions, plus a number of trial repetitions necessary for choosing the correct direction for the next step, given the probability of an incorrect step. Investigated also were several variations of the step system and systems with extrapolation for normally and equally distributed noise. One result of the investigation was that the trial repetitions are either unnecessary or very little necessary, since increases of the average time due to incorrect operations were, in the cases considered, always compensated for almost completely by the time not taken up by the trials which weren't made.

In a communication he presented, Sun Tszian' (Moscow) stated that the statistical method of analyzing automatic scanning processes may play an essential role in the investigation of discrete automatic scanning systems, particularly for those systems which contain complex logical portions. The methods in use up to now, which are based on spectral theory, cannot give a complete probability representation of the convergence, and of other characteristics, of the scanning process. Further development of the statistical method of investigation which was discussed in A. A. Fel'dbaum's paper may lead in the direction of a solution to the following problems: to determine the noise stability of a scanning system as a whole; to determine the probability that a system, exposed to noise of a definite type, will arrive at an extremum after a determinate number of steps; to determine the optimal scanning strategy whereby the probability of arriving at an extremum will be a maximum for a given type and level of noise. The author obtained satisfactory solutions for several of the problems just formulated.

The paper of R. I. Stakhovskii (Moscow) contained a comparison of several scanning methods for automatic optimizers. The author considered the Gauss-Seidel method of alternate changes, the gradient method, and the method of steepest descent. The comparison was made from the point of view of minimum average scan time and increase of accuracy in the extrema determined. It was proven that, for a system which moves by steps, the best scanning algorithm should be taken as a combination of the methods of steepest descent and gradients.

The paper of R. V. Kornilov and N. T. Khristoforov (Moscow) was devoted to the description of an electronic extremal regulator which retains ("memorizes") the extrema, together with an analysis of its operation. Laboratory testing of the system showed that its operation was stable with objects having significant inertia, and also with intense external perturbations of a monotonic character present. The device may be easily coupled with industrial electronic regulators.

M. G. Eskin (Moscow), in his paper, considered an extremal control system for turbine boring with discrete devices. The optimum regimen of the turbine boring is established by so choosing the axial force on the gouge that maximum boring speed is obtained. The absence of any direct means for measuring the face parameters leads to the necessity of indirect measurement of these parameters. To do this, one must take into account the dynamic state of the column of boring tubing and the columnar friction against the well shaft (random noise). The relative slowness of the departure from extrema suggests a two-stage control system. An ordinary control

system maintains a given established load on the gouge. This "given" magnitude of load is established by an extremal regulator. The paper considered relay and proportional systems of extremal control with shutdown in extremal zones and self-starting with departures from extremal zones. In the conclusion, the paper contained a description of the block schematic of a circuit with discrete elements, as well as the results of an investigation into devices using a special dynamic model of a turbine borer.

In the communication of N. V. Grishko (Moscow) there was formulated a criterion for the quality of extremal systems in given statistical conditions which are characterized by disturbances of the extremum states and by high-frequency and low-frequency noise. Results were given of an experimental investigation of sampled-data systems, optimal in the sense of the given criterion, with decaying extrema.

The work of the section on optimal system theory was devoted to the consideration of questions of constructing automatic control systems which are optimal in respect to their rapidity of action.

The paper of A. G. Butkovskii and S. M. Domanitskii (Moscow) presented the results of work on the creation of optimal control systems for objects with delays in the case when the aim of the system is to work off initial mismatches. The authors succeeded in constructing a control system with a very simple structure. The system was used for the automatic matching of the flying blade of a rolling mill with the motion of the rolled strip. As shown by investigation, the system's accuracy was completely satisfactory for practical purposes.

The paper of G. A. Nadzhafova (Moscow) showed ways for determining the limiting rapidity of action of a control system for objects with limitations up to the second order. The results were obtained by plotting the limits of the isochronous domain. This rendered it possible to limit the domain of object states from which objects could be transferred to given states with defined boundaries.

O. G. Varshavskii (Moscow), in his paper, presented the results of the first steps in the work of constructing a general industrial controller for objects with delays, where the control process is close to an optimal one. An analytical investigation and a comparative study of the dynamic properties of ordinary industrial controllers and a controller of the suggested structure showed the essential superiority of the latter with respect to the speed with which the control process was accomplished.

The communication of A. A. Pavlov (Moscow) was devoted to a new method of analyzing optimal systems, illustrated by the example of the synthesis of a control system for an object with third-order boundaries. The special feature of the author's approach to solving the problem is his use of plane movable switching surfaces for the formation of the optimal control laws.

There were 15 papers in the section on sampled-data and digital elements and devices. Considered in the papers were questions of the automation of complex technological processes entailing a large amount of computation, and also questions of creating simplified special-purpose computers, devices for discontinuous control, and elements for computers for a broad class of automatic devices.

In his paper, A. G. Moskalev (Moscow) dealt with the problem of the automatic control of the working regimen of an energy system both by frequency and by active power so as to provide the economically best working regimen for maintaining the frequency at a given level, and also for providing high reliability both in normal and in emergency regimens. The author spoke of two control systems which were implemented and tested in actual working conditions during 1956 and 1957 in the energy systems of the Kherson and Khar'kov Councils of National Economy. Automatic control of the working regimen was implemented on the majority of generating stations, with a nominal power between 80% and 85% of all the nominal power of the energy systems. The experiments provided good results.

The paper of O. M. Kryzhanovskii and M. Yu. Beiderman (Kiev) was devoted to automatic control systems for mines' multicable hoists which provide both accident-free and maximally productive machine operation. The paper gave particular attention to the development of indicators for deep mines and to measuring elements for the control systems. Both devices were implemented as sampled-data devices, and operated with high accuracy and reliability.

The paper of S. M. Khlytchiev and G. A. Aleksandrov (Moscow) dealt with questions concerning the automation of radio receivers in a long-line short-wave radio communications system. The authors showed that it is economically most efficient to automate completely the receiving centers. The paper contained block schematics of automation devices based on the use of electronic controlling machines.

The paper of Yu. I. Kaznacheev, N. D. Kolesnikova and Yu. N. Prozorovskii (Moscow) was devoted to velocity electrovacuum instruments, transforming continuous data to discrete, and vice versa, for broad-band message transmission systems with pulse-code modulation. The authors described coding and regenerating devices constructed on the basis of electron-beam tubes.

B. L. Ermilov (Leningrad) gave a paper on digital analogs of analog computers, these digital computers being sufficiently compact, maintaining the high accuracy of digital computers while combining this with the speed of analog devices. The special features of these digital analogs were shown by the problem of controlling a milling machine.

In his paper, A. K. Zavolokin (Moscow) presented a classification of transformers of continuous quantities into digital equivalents. On the basis of this classification, the author presented a method for obtaining digital equivalents which characterizes, with sufficient definiteness, such important considerations for applications as rapidity of action, noise-resistance, the cyclical nature of the operation, and the unambiguity of null-point definition. The paper treated in detail groups of transformers of serial counters, counter transformers and digit-by-digit coding transformers.

The paper of M. A. Shnaidman (Moscow) was devoted to the investigation of a pulsed-data control system by means of an analog computer. The problem of creating means for simulating pulsed-data systems leads, essentially, to the construction of pulsed-data blocks and their connecting with dc analog devices, to be manufactured by industry, which contain both linear and nonlinear elements. The paper considered circuits for blocks which could provide any form of pulse modulation, these blocks being based on standard operational amplifiers which allowed very high-quality characteristics to be obtained from them.

In the paper of E. L. Itskovich (Leningrad) there was considered a discontinuous-type universal correcting device for implementing a given control of an aggregate's output parameters by varying the regimen by which its operations are stabilized by means of stimuli to the regulator transducers of the stabilizing circuit. The author provided a method for designing correcting devices so as to insure stability and quantitative control of the aggregate parameters under consideration and for creating the conditions of autonomy for other of its parameters. The author cited positive results from the application of the correcting devices developed in a circuit for regulating the process of roasting cement in a rotating kiln.

The paper of V. A. Oleinikov (Leningrad) was devoted to the operation of an electromachine amplifier (EMA) with transverse poles in a sampled-data mode of operation. The author showed that the introduction of the sampled-data mode of operation, due to the vibrational linearization of the nonlinearities, significantly improved the free-running characteristics and the external characteristics of the EMA. The hysteresis loops of the characteristics cited were decreased by a factor between 2 and 3, and this improved the linearity of the characteristics themselves.

The paper contained recommendations on the choice of frequency and pulse length for guaranteeing that the EMA would function within given limits of accuracy.

The paper of B. M. Kagan, V. M. Dolkart, A. I. Voitelev and B. P. Brudnii (Moscow) described a complex universal discrete-continuous machine for the investigation of the processes in control systems controlled by digital machines. The setup consisted of a whole bank of machines.

In the paper of Z. B. Vartanov (Moscow) there was considered the circuit for a discrete machine for controlling a blooming mill. The apparatus consists of a digital servo system of the position type in which a static program is introduced by means of a memory device.

The paper of V. A. Brik (Moscow) investigated the possibilities of constructing simple computers to carry out programs for controlling machines for piece-working, where the profiles of the pieces consist of straight lines and circular arcs. The paper gave special attention to methods for most simply shaping circular arcs. The author showed that the operation of multiplication was not necessary for this sort of program. This significantly simplifies the computer, shortens computation time, and reduces computational error to a minimum.

The paper of G. G. Men'shikov (Leningrad) was devoted to a planned digital computer for computing linear combinations of functions, where the methods used did not include multiplication. The machine for solving the problems formulated would not only lack the operation of multiplication, but would also shorten the total number of operations. The author provided results of experimental testing of this method.

The paper of Ya. E. Gel'fand (Leningrad) considered a device for discontinuous control, with dependent intervals, for cement-producing aggregates.

The very slow flow of the processes and the large lags in the object, sometimes as much as tens of minutes, made it preferable to introduce a discontinuous mode of operation into the controller. The best qualitative control is obtained by introducing a variable control interval as a function of the deviation of the controlled quantity. The paper contained a description of the developed controlling device, currently being produced in small quantities.

In his paper, I. O. Oskolkov (Moscow) presented a method for the multipoint inspection of parameter deviations from given levels by the use of electron-tube indicators for the simultaneous measurement of the magnitudes of the deviations of a large number of parameters. The author considered the possibilities of implementing two-position and three-position control. His paper was based on material obtained from testing of a developed system.

The paper of V. M. Eigenbrot (Moscow) was devoted to a discrete three-position regulator of twenty parameters by one regulator where the objects have large time constants and are self-aligning. The author answered questions concerning the practical building of the circuit, the choice and disposition of the equipment, the method of adjusting the regulator, and he provided results of industrial testing.

In the section of finite automata theory there were considered several questions dealing with the theory of automatic systems which consist of finite numbers of elements, each of which can have a finite number of states.

M. A. Gavrilov (Moscow), in his paper, formulated the problem of constructing complex discrete remote control systems and showed that, by their functions, these systems fall in the class of multiprogram finite automata with discrete feedback loops and with the selection of the program depending both on external stimuli and on the results of the operations of the system itself. The author also showed that further development of discrete remote control systems leads to self-organizing systems.

The paper of P. P. Parkhomenko (Moscow) presented the underlying principles and the mechanisms of a special logical machine for the analysis of relay devices. The machine carries out structural analyses of planned or already constructed relay systems by the method of decomposing the circuits into constituent units, and derives from these decompositions the sequences of operations of the circuit elements, the results being taken off either manually or automatically.

The paper of Yu. Ya. Bazilevskii (Moscow) considered finite determinate automata with constant structures which represented arbitrary occurrences of several arguments. It was shown by the author that from the calculus of temporal logical functions one may form a system of equations and may determine the properties of the system of automata states. The author considered the relationships between abstract automata and the families of subautomata derivable from them, gave a method for decomposing automata into nets with independent nuclei and presented several forms of equivalent transformations and a method for minimizing the matrix of states. The second part of the paper dealt with a method for solving generalized temporal logical equations in implicit form.

In the paper of G. K. Berends and A. A. Talia (Moscow) on pneumatic element systems of finite automata, there was given a description of logical elements "not," "and," and "or," of elements which realize the logical functions of "implication" and "equivalence" and also of delay and repetition elements. Based on these pneumatic elements, devices of industrial importance were developed: regulators with automatic crossover, semiproportional regulators, and extremal regulators with several regulating organs.

The communication of V. N. Roginskii was devoted to a description of a machine for the synthesis of relay device structure. This machine is completely automated and deduces the structure of a device from a switching table for it, given in graphic form, the machine simultaneously determining the required number of additional elements.

The majority of the papers presented at the conference gave rise to lively discussions. While the theoretical level of the papers presented was generally high, the degree of technological development was inadequate. Representatives of institutional branches spoke of their developments in the domains considered, and of the results of testing in use.

**At the concluding session, the section leaders briefly addressed the entire conference, and also formulated the essential problems for further development in the theory and application of discrete automatic systems.**

**I. S. Morosanov and P. I. Chinaev**

# INFORMATION

## FIRST IFAC CONGRESS

### Plans for the Scientific Program of the First Congress of the International Federation for Automatic Control (IFAC) in Moscow

The plans were worked out and presented by the National Committee of the USSR for Automatic Control, and approved by the Executive Committee of IFAC at the meeting in Zurich in March, 1958.

### 1. General Characteristics of the Congress

The first international congress of IFAC was called by a resolution of the General Assembly of IFAC which was passed in Paris on September 12, 1957. Tentatively, the total number of congress participants was set at 1500.

The congress must include three types of scientific and engineering activities in the field of automation.

Branch 1. Theory and methods of automatic regulation and control.

Branch 2. Technological means for automation (switching and computing devices) and the problems related to their creation on new scientific bases.

Branch 3. Industrial applications of automatic regulation and control (including the use of computing devices).

The watchwords of the congress should be "Theory – practical application, technological means – maximum reliability, automation of production – high effectiveness."

It is assumed that the congress will last 10 days and will include: a) the reading and discussion of scientific papers; b) excursions to scientific institutions and also to various undertakings equipped with automatic devices; and, c) visits to the cultural centers of the Soviet Union.

It is assumed that a special program will be set up for the excursions and visits to the Moscow cultural centers for participants and guests of the congress.

### 2. Scientific Program of the Congress

Because, as mentioned, there are three major forms of scientific and engineering activity in the field of automation, the scientific program of the congress will be given in three sections. The program contains an enumeration of the scientific problems and the objects of investigation. Both types of enumeration are presented with a broad brush and do not necessarily exhaust the themes of the congress.

It is assumed that in the first section there will be considered general and particular theoretical problems in the investigation and design of automatic regulation and control systems, as well as methods of solving these problems. Any general or special mathematical posing of a problem and any (exact or approximate) method of solving it will be admissible.

The range of investigations in the theoretical domain will include both abstract problems, which are posed with the aim of finding new methods of solving them, and problems with concrete physical content, which will specify the results obtained which are of scientific or practical value and their suitability for use in engineering

calculations. The objects of investigation may be open-looped or closed-looped systems, with one or several inputs and outputs, with constant parameters, deterministic or stochastic, as well as systems which contain humans or computers as links.

In the second section, the objects of investigation are almost always elementary components of automation systems: transducers, transforming devices, controllers and executive organs. For the entire set of these devices it is assumed that there will be scientific papers on the theory and design of transducers as measuring instruments, on the theory and design of devices which transform one physical quantity into another and, finally, the theory and design of controllers and executive mechanisms.

The general problem of this theory and design reduces to the determination of exact data on these elements, and also the determination of their static and dynamic characteristics. This same problem also pertains to any set of elements of technological means which form a single system for automatic inspection, regulation, and control, or of computing devices. Also relevant here is the most important problem of technological means, the problem of maximum reliability. The presentation of solutions of these problems can be illustrated by the results of experimental study of the technological means, and also by generalizations of their trials in use.

The same section must include descriptions of the principles of concocting technological means, illuminated by trials of their design, preparation, and questions of economy. Here also should be presented new physical principles on which may be based the development and design of new instruments, automation systems, and inspecting and computing devices.

An important place in the third section is filled by theoretical and experimental methods for studying the static and dynamic properties of objects and technological processes, and also methods for the analysis of disturbing stimuli.

All these data, together with the mathematical descriptions of the objects and technological processes to be automated, as well as with generalizations from the empirical testing of these objects, will be used in formulating the scientific bases of automatic production. On this basis there may be presented the design principles of automatic inspection and control systems, automated complexes, and also the bases of the automatic control of production flow. Also included in this section should be new regulation and control principles.

The final problem is that of improving the conditions, and increasing the productivity, of work, as well as the economic efficiency of automation. The objects of investigation here are either the processes to be automated, grouped by some inherent physical or other principle of their action, or the automated objects.

In every section the problem of the historical investigation of the area of automation, as well as the problem of creating a single international terminology, will be discussed.

The subject matter of a paper can be any scientific or technological question given on our program, or close to it. Below we give a list of the scientific problems and objects relating to them in all three sections of the program.*

Section 1. Theory. General and Particular Problems in Automatic Regulation and Control Theory.

1) Approximations to, and simplifications of, design methods for controllers and for control systems.

2) Methods for determining optimal adjustments of automatic controllers.

3) Methods for finding the design characteristics of a system based on the use of experimental data.

4) Analyses of concrete control systems and generalization of their results.

5) Applied methods of synthesizing optimal systems.

6) Nomograms, tables, graphs, slide rules, and other subsidiary aids for the analysis and synthesis of control systems.

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*For example, problem 1, "Approximations to, and simplifications of, design methods for controllers and for control systems," is relevant to all the objects from 1 to 10 listed under "Objects of investigation" in the given Section, etc.

7) The stability problem.

8) Periodic motion problems.

9) The problem of estimating the quality of the transient response, as well as constructing it. The theory of sliding motion.

10) The problems of structure synthesis and parameter selection.

11) The problem of optimal regulation and control.

12) The problem of programmed regulation and control.

13) The problem of invariance and the theory of combined systems.

14) The statistical theory of automatic regulation and control systems.

15) Programming theory.

16) The theory of operation and logic diagrams.

17) The theory of the transmission of remote control signals.

18) Simulation and machine synthesis.

19) The statics of controlled systems and the computation of their steady states.

20) Terminology and history.

Objects of Investigation.

1) Continuous systems (analog systems).

2) Systems with discontinuous characteristics.

3) Pulsed (sampled-data) systems.

4) Regulation and control systems with computing devices.

5) Self-adjusting (self-organizing) systems.

6) Systems of extremal control.

7) Servo systems.

8) Prediction systems.

9) Systems with programmed control.

10) Systems with combined forms of operation.

Section 2. Technological Means. The Problems and Theory of the Technological Means of Automatic Control.

1) Theory and computational methods for the static and dynamic characteristics of the elements of automatic regulation and control systems.

2) Methods for the experimental investigation of the properties of elements and systems of automatic inspection, regulation, and control.

3) The analysis of disturbing stimuli.

4) Trials in use.

5) Classification, principles, and the theoretical bases of designs for elements and systems of automatic inspection, regulation, and control.

6) The design of technological means.

7) The technology of technological means.

8) The economics of technological means.

9) New physical principles.

10) Terminology and history.

Objects of Investigation.

1) Transducers and transformers.

2) Controllers and their elements.

3) Executive organs.

4) Systems of automatic inspection, regulation, and control.

5) Systems of remote regulation and control.

6) Controlling machines.

Section 3. The Automation of Production. Problems of Control Theory.

1) Methods of investigating and designing the static and dynamic characteristics of the objects of regulation and control of technological processes.

2) Methods for the experimental investigation of the properties of objects of regulation and control, and of technological processes.

3) Analysis of disturbing stimuli.

4) The classification and mathematical description of the objects of control and of technological processes.

5) Trials in use.

6) Principles and theoretical basis of designing automatic regulation and control systems.

7) Automatic control of production flow.

8) The development of automatic complexes (automatic factories, plants, etc).

9) New principles of regulation and control.

10) The productivity and economic effectiveness of automation.

11) Terminology and history.

Objects of Investigation.

1) Electrical machines: generators, motors, transformers.

2) Prime movers: turbines, motors, steam machines.

3) Thermotechnological machines: furnaces, steam boilers, reactors.

4) Chemical processes.

5) Metallurgical processes: reducing processes, steel working, nonferrous metal production, alloys.

6) Electrometallurgical processes: electrolysis, plating.

7) Hydrometallurgical processes.

8) Assembly-line material flow: rolling mills, paper-making machines, textile finishing processes.

9) Forging and construction.

10) Transport: railroads, cranes, rail-less wheeled transport.

11) Flying and sea-going apparatus: airplanes, ships, boats, rockets, torpedoes.

12) Mechanical processes for material processing: mills, presses, hammers.

13) Processes for handling freely-flowing material: concrete, plastic mixtures, ore concentration, neutralization and filling of ores.

14) Mining and extracting operations: oil drilling, water, gas, oil and gas piping, water piping.

## 3. Procedures for Preparation and Selection of Papers

It is proposed to establish the following procedures for the selection and preparation of papers for the congress.

1) In each country, the National Committee must make the selection of papers submitted for presentation at the congress.

2) The papers must contain scientific (theoretical or experimental) or practical results of the work of one or a group of authors in any of the enumerated domains, or in a new domain related to those given in Section 2 above, as well as subjects related to the development of methods of designing automatic systems, or experience in their industrial use.

3) The National Committees will send the papers selected by them for presentation at the congress to the Executive Committee of IFAC.

4) All papers submitted by the National Committees for presentation at the congress and approved by the Executive Committee of IFAC will be published in the Proceedings of the Congress in the English and Russian languages.

5) Individual scientists and engineers of all countries may take part in the congress and present papers without submitting them to their National Committees.

However, special permission of the Executive Committee of IFAC is necessary for all such papers.

6) It is desirable that the size of typed page be not greater than half a printed page, including illustrations.

7) Papers are to be given either in Russian or in English.

## 4. Procedures for Preparing and Conducting the Congress

1) The congress opens June 23, 1960. Duration of the congress will be ten days.

2) The deadline for the submission of papers by the National Committee of the Soviet Union is October 31, 1959.

3) Papers prepared for printing and for authors' corrections by December 31, 1959.

4) Printed by March 31, 1960.

5) Distribution of papers to the authors and participants in the congress on April 30, 1960.

6) Over-all congress schedule.

The first day will be devoted to the plenary session, which will include: a) the opening of the congress; b) a paper on the subject, "The science of automatic control and the development of technology."

The following days, from the second to the ninth, inclusive, will be given over to the work of the Sections.

On the final day there will be a plenary session devoted to reports of the Section leaders, current Federation matters, the passing of resolutions, and the presenting of various addresses.

7) The dividing up of the congress into Sections must be carried out after the papers have been obtained, and will be dictated by the scientific content of these papers.

## Instructions for Congress Participants

The National Committee of the USSR on Automatic Control is now accepting applications for the presenting of papers at the First Congress of IFAC.

All those desirous of presenting papers should submit their applications by February 15, 1959 to the National Committee: Moscow, Kalanchevskaia 15a, Institute for Automation and Remote Control, AN SSSR. The applications must contain the title of the paper and its abstract (without illustrations), taking no more than three typewritten pages.

The complete text of the papers must be submitted to the National Committee no later than May 15,1959.*

The papers must be prepared in accordance with the rules for preparation of papers for publication in the journal, "Automation and Remote Control" (USSR) as given in the issue of No. 1, 1959. If desired, additions to the papers, suitable for discussion purposes, may be prepared by the participants and sent to the National Committee of the USSR no later than May 15, 1960.

The National Committee of the USSR for Automatic Control requests all organizations and authors to present their papers, if at all possible, both in Russian and in English.

All papers approved by the National Committee will be published in the Proceedings of the Congress; all papers approved by the Executive Committee of IFAC will be read at the congress as bases for discussions on the given questions.

In order that the discussion have maximum scientific value, these papers will be printed and distributed to the delegates** to the Congress within two months of closing.

The Presidium of the National Committee
of the USSR for Automatic Control

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*The deadline for submission of papers was dictated by the necessity of translating them into English, reviewing them, and submitting them to the Executive Committee of IFAC.

**The formation of the delegation from the USSR to the congress will be implemented subsequently.

# ERRATA

## Volume 19, Number 10, 1958

| Page | Line | Reads | Should read |
|---|---|---|---|
| 939 | Eq. (24) | $-r \leq \xi < x(t)$ | $-r \leq \underline{\xi} < x(t)$ |
| 945 | Line 2 | OX axis, | $\overline{OX}$ axis, |
| " | Line 3 | $F^{(N)}(\bar{\omega}'_{S_2})$ $\omega'_{S_2}$ | $F^{(N)}(\bar{\omega}'_{S_2}) \subset \bar{\omega}_{S_2}$ |
| " | Footnote | $F^{(N)}(\bar{\omega}_{\alpha})$ $\bar{\omega}_{\alpha}$, | $F^{(N)}(\bar{\omega}_{\alpha}) \subset \bar{\omega}_{\alpha}$, |